Beginning Doctoral Program Gerzensee

Lectures held by Klaus Schmidt and Piero Gottardi

Microeconomics Midterm Solutions

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1 Microeconomics Midterm 2011 / 12

Schmidt

Exercise 1

(a) Yes since

$$\begin{aligned} x_k(\lambda p, \lambda w) &= \frac{\lambda w}{\sum_{e=1}^{c} \lambda p_e} = \frac{\lambda}{\lambda} \frac{w}{\sum_{l=1}^{\infty} p_e} \\ &= \frac{w}{\sum_{e=1}^{l} p_e} = x_k(p, w) \end{aligned}$$

(b) Yes since

$$\begin{aligned} \sum_{k=1}^{L} x_k p_k &= \sum_{k=1}^{L} \frac{w}{\sum_{\ell=1}^{w} p_e} p_k = \frac{w}{\sum_{\ell=1}^{L} p_e} \sum_{k=1}^{L} p_k \\ &= w \frac{\sum_{k=1}^{L} p_k}{\sum_{\ell=1}^{L} p_e} = w \end{aligned}$$

(c) Yes since WA says that

$$p \times (p', w') \leqslant w \Longrightarrow p' \times (p, w) > w'$$

In our case:

$$\underbrace{w' \frac{\sum_{\ell=1}^{L} p_l}{\sum_{\ell=1}^{L} p_l'} \leqslant w}_{\text{(I)}} \Rightarrow \underbrace{w \frac{\sum_{\ell=1}^{L} p_e'}{\sum_{\ell=1}^{L} p_e} > w'}_{\text{(II)}}$$

From (I) and $x(p, w) \neq x(p', w')$ implies (II). Thus, WA is satisfied!

(d)

$$
\begin{aligned}
s_{lk}(p,w) &= \frac{\partial x_k(p,w)}{\partial p_k} + \frac{\partial x_l(p,w)}{\partial w} x_k(p,w) \\
&= -\frac{w}{\left(\sum_{l=1}^{L} p_l\right)^2} + \frac{w}{\left(\sum_{l=1}^{L} p_l\right)^2} = 0
\end{aligned}
$$

Since all entries are zero it is symmetric and negative semidefinite.

Exercise 2

(a) This is immediate. Since preferences are represented by $f(x) = g(h(x))$, they are also represented by $h(x)$ as utility is only ordinal.

$$
\begin{aligned}
x > y &\Longleftrightarrow f(x) > f(y) \quad \text{by utility function} \\
f(x) > f(y) &\Longleftrightarrow h(x) > h(y) \quad \text{by monotonic transformation}
\end{aligned}
$$

(b) $e(p,u)$ is the answer to

$$
\min_x px \text{ s.t. } u(x) = u
$$

(1) Let $u(x) = 1$, and x^* the solution:

$$
\begin{aligned}
&\min_x px \text{ s.t. } u(x) = 1 \\
&\longrightarrow x^* = \operatorname{argmin}(px) \\
&\longrightarrow u\left(x^*\right) = 1
\end{aligned}
$$

Gottardi

Exercise 1

(a) Agent h:

$$\max_{x^h} \ln\left(x_1^h\right) + k^h \ln\left(x_2^h\right)$$
$$\text{s.t. } px_1^h + x_2^h = pw_1^h + w_2^h$$

First order conditions:

$$\frac{1}{x^\mu} - \lambda p = 0$$
$$\frac{k^n}{x_2^h} - \lambda = 0$$
$$\Rightarrow x_2^h = k^h p x_1^h \tag{1}$$

Plug (1) into BC for A:

$$px_1^A + 3px_1^A = p13 \Longleftrightarrow x_1^A = \frac{13}{4}$$

Plug (1) into BC for B:

$$px_1^B + px_1^B = 14 \Longleftrightarrow x_1^3 = \frac{7}{p}$$

Market clearing:

$$
\begin{aligned}
x_1^B = 13 - x_1^A = \frac{3.13}{4} = \frac{39}{4} \to \frac{39}{4} &= \frac{7}{p} \\
\iff p = \frac{4 \cdot 7}{39} &= \frac{28}{39} \\
x_2^A = 3 \cdot p \cdot x_1^A = 3\frac{28}{35}\frac{13}{4} = \frac{7 \cdot 13}{13} &= 7 \\
x_2^B &= 7
\end{aligned}
$$

Competitive Equilibrium:

$$
\begin{aligned}
\left(x_1^A, x_2^A\right) &= (\frac{13}{4}, 7) \\
\left(x_1^B, x_2^B\right) &= (\frac{39}{4}, 7) \\
p &= \frac{28}{39}
\end{aligned}
$$

(b) Yes.

$$
\begin{aligned}
MRS^A &= \frac{x_2^A}{3x_1^A} = \frac{7}{3 \cdot \frac{13}{4}} = \frac{28}{39} \\
MRS^B &= \frac{x_2^B}{x_1^B} = \frac{7}{\frac{39}{4}} = \frac{28}{39}
\end{aligned}
$$

Also: markets are complete, there's free disposal, and LNS is satisfied.

(c) Yes.

$$
\begin{aligned}
\mathrm{MRS}^A(4, 8) &= \frac{8}{3 \cdot 4} = \frac{2}{3} \\
\mathrm{MRS}^B(9, 6) &= \frac{6}{9} = \frac{2}{3}
\end{aligned}
$$

As preferences are convex, we can decentralize:

$$T^A = \begin{bmatrix} x_1^A - w_1^A \\ x_2^A - w_2^A \end{bmatrix} = \begin{bmatrix} 4 - 13 \\ 8 \end{bmatrix} = \begin{bmatrix} -9 \\ 8 \end{bmatrix}$$

$$T^B = \begin{bmatrix} x_1^B - w_1^B \\ x_2^B - w_2^B \end{bmatrix} = \begin{bmatrix} 9 \\ 6 - 14 \end{bmatrix} = \begin{bmatrix} 9 \\ -8 \end{bmatrix}$$

At equilibrium, relative price must be equal to MRS. Thus $p = \frac{2}{3}$.

Exercise 2

(a) at $t = 0:$ $\quad q_1\theta_1 + c_2\theta_2 = 0$

at $t = 0 : s = 1 :$ $\quad x_1 = w_1 + 3\theta_1 + \theta_2 = 10 + 3\theta_1 + \theta_2$

at $t = 0 : s = 2 :$ $\quad x_2 = w_2 + \theta_1 + 3\theta_2 = 4 + \theta_1 + 3\theta_2$

(b) We solve the consumer problem:

$$\max_x \frac{1}{2}\left[\ln(x_1) + \ln(x_2)\right] \text{ s.t. BCs from (a)}$$

substitute (x_1, x_2) from the BC in (a):

$$\max_\theta \frac{1}{2}\left[\ln(10 + 3\theta_1 + \theta_2) + \ln(4 + \theta_1 + 3\theta_2)\right]$$
$$\text{s.t. } q_1\theta_1 + q_2\theta_2 = 0$$

First order conditions for $(\theta_1, \theta_2, \lambda)$:

$$
\begin{aligned}
\frac{1}{2}\left[\frac{3}{10+3\theta_1+\theta_2}+\frac{1}{4+\theta_1+3\theta_2}\right]-\lambda q_1 &= 0 \\
\frac{1}{2}\left[\frac{1}{10+3\theta_1+\theta_2}+\frac{3}{4+\theta_1+3\theta_2}\right]-\lambda q_2 &= 0 \\
q_1\theta_1+q_2\theta_2 &= 0
\end{aligned}
$$

Let $q_1 = q_2 = 0 \to \theta_1 = -\theta_2$ then

$$
\begin{aligned}
\frac{3}{10+3\theta_1+\theta_2}+\frac{1}{4+\theta_1+3\theta_2} &= \frac{1}{10+3\theta_1+\theta_2}+\frac{3}{4+\theta_1+3\theta_2} \\
3\left[4+\theta_1+3\theta_2\right]+1\left[10+3\theta_1+\theta_2\right] &= 1\left[4+\theta_1+3\theta_2\right]+3\left[10+3\theta_1+\theta_2\right] \\
2\left[4-2\theta_1\right] &= 2\left[10+2\theta_1\right] \\
-6 &= 4\theta_1 \\
\theta_1 &= -\frac{3}{2}
\end{aligned}
$$

As $\theta_1 \neq 0$, this is not a CE. There is only one consumer and if $\theta_1 \neq 0$, then there is excess supply or demand!

(c) The consumer is poorer in state 2. Thus, he wants to insure against it as he is risk-averse by the concavity of utility. This drives up the price of asset 2 compared to asset 1. Thus $q_2 > q_1$.

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Schmidt

Exercise 1

(a) To violate WA, both bundles must be affordable under both price-wealth-situations:

$$\left|\begin{array}{c} 540 \leqslant 360 + 24x \\ 30(12 + x) \leqslant 600 \end{array}\right|$$

$$\Leftrightarrow \quad \left|\begin{array}{c} 7.5 \leq x \\ x \leq 8 \end{array}\right|$$

WA is violated when $x \in [7.5, 8]$

(b) Bundle 2 must be affordable in period 1: $x \leq 8$. Thus, the consumer prefers bundle 1 to 2 when $x \in [0, 7.5)$.

(c) I think he means good 2.

As price decreased, we must have a decrease in consumption to satisfy $\frac{\partial x_\ell}{\partial p_\ell} > 0$.

Thus: $x < 10$

In order to not violate WA, we are left with $x \in [0, 7.5) \cup (8, 10)$.

Exercise 2

(a) Let $f(\cdot)$ be a monotonic transformation and apply Roy's identity to $f(v(p, w))$:

$$\tilde{x}_\ell(p,w) = -\frac{\frac{\partial f(v(p,w))}{\partial p_\ell}}{\frac{\partial f(v(p,w))}{\partial w}} = -\frac{\frac{\partial f(v(p,w))}{\partial v(p,w)} \cdot \frac{\partial v(p,w)}{\partial p_\ell}}{\frac{\partial f(p_p,w))}{\partial v(p,w)} \frac{\partial v(p)}{\partial w}} = -\frac{\frac{\partial v(p,w)}{\partial p_\ell}}{\frac{\partial v(p)}{\partial w}} = x_\ell(p,w)$$

Even by implementing $f(\cdot)$ we find the same $x_\ell(p,w)$.

(b) (1) Invert $v(p,w)$ to find $e(p,u)$:

$$e(p,u) = u\left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha}$$

(2) Apply Shepherd's Lemma:

$$\begin{aligned} h_1(p,u) = \frac{\partial e\left(p_1 u\right)}{\partial p_1} &= u\alpha^{-\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha}\alpha p_1^{\alpha-1} \\ &= u\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{p_2}{p_1}\right)^{1-\alpha} \end{aligned}$$

(c)

case 1: $\alpha = \alpha\left(\frac{p_1}{p_2}\right)$

$$u_1\left(\lambda p, u\right) = u\left[\frac{\alpha\left(\frac{\lambda p_1}{\lambda p_2}\right)}{1-\alpha\left(\frac{\lambda p_1}{\lambda p_1}\right)}\frac{\lambda p_2}{\lambda p_1}\right]^{1-\alpha\left(\frac{\lambda p_1}{\lambda p_2}\right)} = u\left(\frac{\alpha\left(\frac{p_1}{p_2}\right)}{1-\alpha\left(\frac{p_1}{p_2}\right)}\frac{p_2}{p_1}\right)^{1-\alpha\left(\frac{p_1}{p_2}\right)} = h_1\left(p_1 u\right)$$

case 2: $\alpha = \alpha\left(p_1\right)$

$$u_1\left(\lambda p, u\right) = u\left[\frac{\alpha\left(\lambda p_1\right)}{1-\alpha\left(\lambda p_1\right)}\frac{\lambda p_2}{\lambda p_1}\right]^{1-\alpha(\lambda p_1)} = u\left[\frac{\alpha\left(\lambda p_1\right)}{1-\alpha\left(\lambda p_1\right)}\frac{p_2}{p_1}\right]^{1-\alpha(\lambda p_1)} \neq h_1\left(p_1 u\right)$$

Exercise 3

As the returns to scale are constant, we must apply cost-minimization.

$$\min_x wx \text{ s.t. } f(x) = 1$$

We differentiate with respect to x_ℓ to find FOC:

$$\begin{aligned}
w_\ell - \lambda \frac{\partial f(x)}{\partial x_\ell} &= 0 \\
w_\ell x_\ell^* - \lambda \frac{\partial f(x)}{\partial x_\ell} x_\ell^* &= 0 \quad \text{use Euler's formula} \\
wx^* - \lambda \sum \frac{\partial f(x)}{\partial x_\ell} x_\ell^* &= 0 \\
wx^* - \lambda \cdot 1 &= 0 \\
wx^* = c(w) &= \lambda
\end{aligned}$$

By constant returns to scale $\min_x wx$ s.t. $f(x) = y$ will give

$$\begin{aligned}
w\tilde{x} - \lambda \sum \frac{\partial f(x)}{\partial x_e} \tilde{x}_e &= 0 \\
w\tilde{x} - \lambda y &= 0 \\
w\tilde{x} = c(w, y) &= \lambda y = c(w) \cdot y
\end{aligned}$$

Gottardi

Exercise 1

(a) Consumer A:

$$\max_{x^A} x_1^A + 2\left(x_2^A\right)^{1/2} \text{ s.t. } px_1^A + x_2^A = p5 \iff \max_{x_2^A} 5 - \frac{x_2^A}{P} + 2\left(x_2^A\right)^{1/2}$$

First Order Condition:

$$\begin{aligned} & -\frac{1}{p} + \left(x_2^A\right)^{-1/2} = 0 \\ \Leftrightarrow \quad & x_2^A = p^2 \rightarrow x_1^A = 5 - p \end{aligned}$$

Consumer B:

$$x_1^B = \begin{cases} \infty & \text{if} \quad p < 2 \\ \mathbb{R}^+ & \text{if} \quad p = 2 \\ 0 & \text{if} \quad p > 2 \end{cases}$$

$$x_2^B = \begin{cases} \infty & \text{if} \quad p > 2 \\ \mathbb{R}^+ & \text{if} \quad p = 2 \\ 0 & \text{if} \quad p < 2 \end{cases}$$

Market Clearing:

$$\begin{aligned} w_1 = 5 = x_1^A + x_1^B = p^2 + x_1^B \\ w_2 = 6 = x_2^A + x_2^B = 5 - p + x_2^B \end{aligned}$$

$p = 2$ must hold. Otherwise excess demand would not be zero, and markets can't clear.

Edgeworth Box:

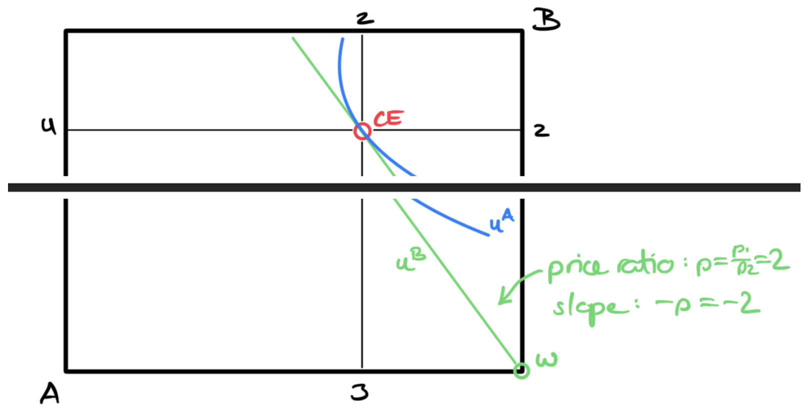


Figure 1: The vertical bar is not part of the figure. The figure was drawn on an iPad and the export created the line. Ignore it.

(b) Agent A cannot influence x_2^B. Thus, her FOC does not change: her behaviour is the same. The behavior of agent B does not change as well. Thus, the CE remains the same. But this CE does not need to be PE anymore, reason being that X_2^B is on externality for A. Incomplete markets lead to inefficient CE allocations.

(c) type C: Under autarky there's no trade as consumers are identical. Free trade can only lead to a utility increase (or it stays the same) by voluntarity of trade.

type C: If the greater total endowment of good 2 in the economy increases p. then A profits as a seller of good 1 . If price remains at $p = 2$, there is no impact.

type B: If $p > 2$, B will not sell anything of good 2, and try to buy more of it (which she cannot). she can't). Then $\left(u^B\right)^{j\text{FT}} = 6 = \left(u^B\right)^{\text{aut}}$. it $p = 2$, also $\left(u^B\right)^{j\text{FT}} = 6 = \left(u^B\right)^{\text{aut}}$.

Exercise 2

Convexity is not needed, but LNS is. Convexity is only needed for the SWT. If LNS is violated, we can immediately construct a counterexample with $L = H = 2$:

Although CE exists, we could move south-west to increase B's utility without hurting A.

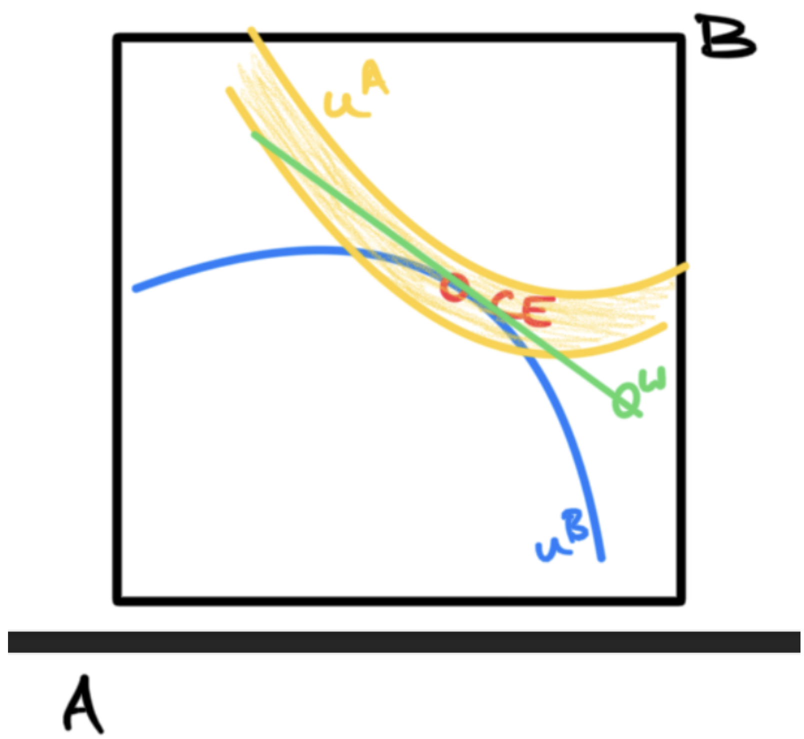


Figure 2: The vertical bar is not part of the figure. The figure was drawn on an iPad and the export created the line. Ignore it.

Exercise 3

(a)

$$w^1 = (3, 2) \quad w^2 = (2, 6)$$

PE: equalize MRS across consumers.

$$MRS^1 = \frac{\pi(1)\frac{1}{x^1(1)}}{\pi(2)\frac{1}{x^1(2)}} \stackrel{!}{=} MRS^2 = \frac{\pi(1)\frac{1}{x^2(1)}}{\pi(2)\frac{1}{x^2(2)}}$$
$$\frac{x^1(2)}{x^1(1)} = \frac{x^2(2)}{x^2(1)}$$

Apply market clearing conditions:

$$x^2(2) = 8 - x^1(2)$$
$$x^2(1) = 5 - x^1(1)$$

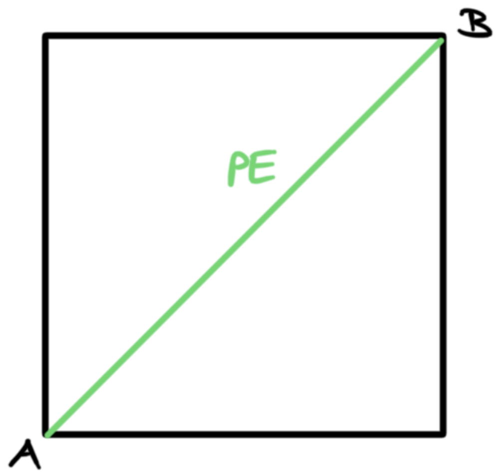
Thus:

$$\frac{x^1(2)}{x^1(1)} = \frac{8 - x^1(2)}{5 - x^1(1)}$$
$$\frac{8}{x^1(2)} - 1 = \frac{5}{x^1(1)} - 1$$
$$x^1(2) = \frac{8}{5}x^1(1)$$

(b) <u>consumer h:</u>

$$\max_{x^h} \pi(1)\ln\left(x^h(1)\right) + \pi(2)\ln\left(x^h(2)\right)$$
$$\text{s.t.} \quad p(1)\left(x^h(1) - w^h(1)\right) + p(2)\left(x^h(2) - w^h(2)\right) = 0$$

The FOCs are



$$
\begin{aligned}
&\frac{\pi(1)}{x^h(1)} - \lambda p(1) = 0 \\
&\frac{\pi(2)}{x^h(2)} - \lambda p(2) = 0 \\
\Longrightarrow \quad &\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)} \frac{x^h(2)}{x^h(1)} \qquad \text{(I)}
\end{aligned}
$$

Equation (I) describes the relationship of prices and state probabilities. With identical beliefs, we have $\frac{x^1(2)}{x^1(1)} = \frac{x^2(2)}{x^2(1)}$, ie. PE and thus

$$
\frac{p(1)}{p(2)} = \frac{\pi(1)}{\pi(2)} \frac{8}{5}
$$

Therefore, in our case we find that $\frac{p(1)}{p(2)} > \frac{\pi(1)}{\pi(2)}$ because total endowment in state 1 is higher than in state 2. If there was greater total endowment in state 1, the inequality sign would switch to $<$.

3 Microeconomics Midterm 2011 / 12

Schmidt

Exercise 1

(a) As all bundles are different, we need to check for affordability of each bundle under each price-wealth-situation. As we see in the table, whenever $p^t \times \left(p^{t'}, w^{t'}\right) \leq w^t$ we have $p^{t'} \times (p^t, \omega^t) > \omega^{t^t}$, and WA holds.

Situation	Bundle	Expenditure	Compare	Conclusion
at (p^0, w^0)	x^1	$p^0 x^1 = 96$	$> w^0$	x^1 not aff.
	x^2	$p^0 x^2 = 80$	$< w^0$	x^2 is aff.
at (p^1, w^1)	x^0	$p^1 x^0 = 33$	$< w^1$	x^1 is aff.
	x^2	$p^1 x^2 = 39$	$> w^1$	x^2 not aff.
at (p^2, w^2)	x^0	$p^2 x^0 = 52$	$> w^2$	x^1 not aff.
	x^2	$p^1 x^2 = 48$	$< w^2$	x^2 is aff.

(b) Whenever multiple bundles are affordable under are price-wealth-situation, we can make observations about revealed preference:

- at $(p^0, w^0) : x^0 \succ x^2$
- at $(p^1, w^1) : x^1 \succ x^0$

  By transitivity we must have $x^1 \succ x^2$. But:
- at $(p^2, w^2) : x^2 \succ x^1$

We have found a violation of transitivity.

Exercise 2

(a) Apply Roy's identity:

$$x_1(p, w) = -\frac{\frac{\partial v(pw)}{\partial p_1}}{\frac{\partial v(pw)}{\partial w}} = -\frac{\frac{w}{p_1^2}}{\frac{1}{p_1} + \frac{1}{p_2}} = \frac{w}{p_1} \frac{1}{1 + \frac{p_1}{p_2}}$$

(b) (1) Invert $v(p,w)$ to find w. At optimum we have $v(p,w)=u; w=e(p,u)$:

$$\begin{aligned}
w = v\left(p,w\right)\frac{1}{\frac{1}{p_1}+\frac{1}{p_2}} &= v\left(p,w\right)\left[\frac{1}{p_1}+\frac{1}{p_2}\right]^{-1}\\
\Longleftrightarrow e\left(p,u\right) = u\left[\frac{1}{p_1}+\frac{1}{p_2}\right]^{-1}
\end{aligned}$$

(2) Apply Shephard's Lemma:

$$\begin{aligned}
h_1(p,u) &= \frac{\partial e\left(p,u\right)}{\partial p_1} = u(-1)\left[\frac{1}{p_1}+\frac{1}{p_2}\right]^{-2}(-1)\frac{1}{p_1^2}\\
&= u\left[1+\frac{p_1}{p_2}\right]^{-2}
\end{aligned}$$

(c) Yes.

$$x_1\left(\lambda p,\lambda w\right) = \frac{\lambda w}{\lambda p_1}\frac{1}{1+\frac{\lambda p_1}{\lambda p_2}} = \frac{w}{p_1}\frac{1}{1+\frac{p_1}{p_2}} = x_1\left(p,w\right)$$

(d) Let $f(\cdot)$ be a monotonic transformation. Then by Roy's identity:

$$\begin{aligned}
\tilde{x}_l(p,w) &= -\frac{\frac{\partial f(v(p,w))}{\partial p_l}}{\frac{\partial f(v(p,w))}{\partial w}} \quad \text{apply chain-rule}\\
&= -\frac{\frac{\partial f(v(p,w))}{\partial v(p,w)}\cdot\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial f(v(p,w)}{\partial v(p,w)}\cdot\frac{\partial v(p,w)}{\partial w}} = -\frac{\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial v(p,w)}{\partial w}} = x_l(p,w)
\end{aligned}$$

Exercise 3

(a) <u>IF:</u>

$$
\begin{aligned}
u(x) &= \alpha + \beta \left(-e^{-cx}\right) \\
u'(x) &= -c\beta \left(-e^{-cx}\right) \\
u''(x) &= -c^2\beta \left(-e^{-cx}\right) \\
r(x) &= -\frac{-c^2\beta e^{-cx}}{c\beta e^{-cx}} = c
\end{aligned}
$$

<u>ONLY IF:</u>

$$
\begin{aligned}
& r(x) = -\frac{u''(x)}{u'(x)} = -\frac{d\ln\left(u'(x)\right)}{dx} = c \\
& \int_{\underline{x}}^{x} \frac{d\ln\left(u'(t)\right)}{dt} dt = -c\int_{\underline{x}}^{x} dt \\
& \ln\left(u'(x)\right) - \ln\left(u'(\underline{x})\right) = -c(x - \underline{x}) \\
& \frac{u'(x)}{u'(\underline{x})} = \exp(-cx)\exp(c\underline{x}) \\
& \int_{\underline{x}}^{x} u'(y)dy = \int_{\underline{x}}^{x} \exp(-cy)dt\exp(c\underline{x})u'(\underline{x}) \\
& u(x) - u(\underline{x}) = -\frac{1}{c}(\exp(-cx) - \exp(-c\underline{x}))\exp(c\underline{x})u'(\underline{x}) \\
& u(x) = u(\underline{x}) - \frac{1}{c}(\exp(-cx) - \exp(-c\underline{x}))\exp(c\underline{x})u'(\underline{x})
\end{aligned}
$$

By choosing α, β, c correctly, we can get:

$$
u(x) = \alpha - \beta\exp(-cx)
$$

(b)

$$
\max_a EU(w - a + az) = \max_a \int -\exp(-c(w - a + az))dF(z)
$$

Obtain the FOC:

$$
\begin{aligned}
\frac{\partial EU(\cdot)}{\partial a} = \int -\exp(-c(w-a+az))(-c)(z-1)dF(z) &\overset{!}{=} 0 \\
c\int \exp(-cw)\exp(ca)\exp(-caz)(z-1)dF(z) &= 0 \\
\underbrace{c \cdot \exp(-cw)\exp(ca)}_{\neq 0} \int \exp(-caz)(z-1)dF(z) &= 0 \\
\int \exp(-\,\mathrm{caz})(z-1)dF(z) &= 0
\end{aligned}
$$

The last line implicitly defines the optimal $\tilde{a}$ and it is independent of w.

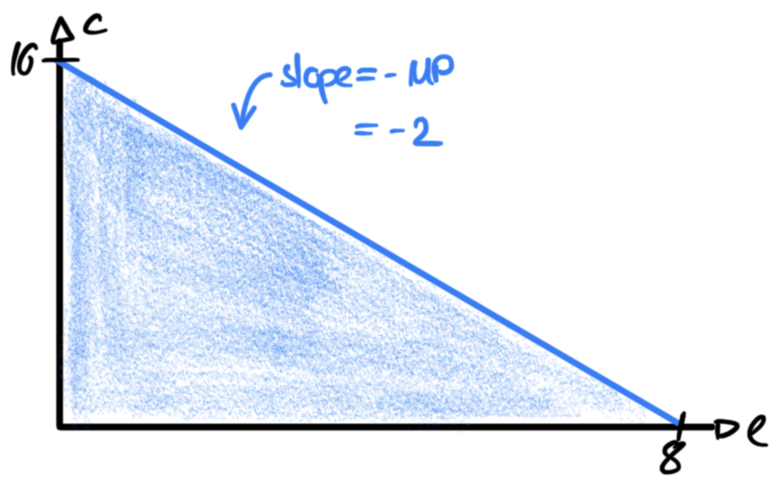
Gottardi

Exercise 1

(a) Maximum production / consumption: $2 \cdot 8 = 16$ Maximum leisure : 8

The blue triangle (incl. border) is feasible. The border is the set of PE allocations, described by

$$c = 16 - 2e$$



(b) Consumer Problem:

$$\max c + l^{1/2}$$
$$\text{s.t. } pc = \omega[8 - l] + \pi$$

FOCs:

$$
\begin{aligned}
1 - \lambda p &= 0 \\
\frac{1}{2} l^{-1/2} - \lambda w &= 0 \\
\implies \quad \frac{1}{2} l^{-1/2} &= \frac{w}{p} \\
\Leftrightarrow \quad l &= \left(\frac{p}{w} \frac{1}{2} \right)^2
\end{aligned}
$$

<u>Firm Problem:</u>

$$
\max_L p2L - wL \Leftrightarrow \max_L L(2p - w)
$$

$$
L = \begin{cases} \infty & \text{if } p/w > 1/2 \\ \mathbb{R}^+ & \text{if } p/w = 1/2 \\ 0 & \text{if } p/w < 1/2 \end{cases}
$$

<u>Market Clearing:</u>

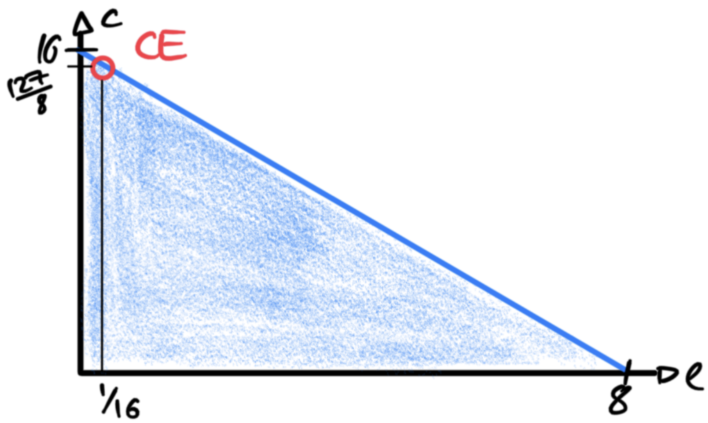
$$
\begin{aligned}
& c = y = 2L \\
& l = 8 - L \\
\implies \quad & c = 2(8 - l)
\end{aligned}
$$

Since any price other than 1/2 would lead to excess demand of one good or the other, set

$$
\begin{aligned}
\frac{p}{w} = 1/2 \longrightarrow l = \frac{1}{16} &\longrightarrow C = \frac{127}{8} = y \\
&\longrightarrow L = \frac{127}{16}
\end{aligned}
$$

Competitive Equilibrium:

$$y = \frac{127}{8}; L = \frac{127}{16}; \frac{p}{w} = \frac{1}{2}$$



(c) This shifts the equilibrium along the PE allocations to more leisure and less consumption. Nothing changes for the firm. Thus: $\frac{p}{w} = \frac{1}{2}$ as before. For the consumer we now have $l = \left(\frac{p}{w}\right)^2 = \frac{1}{4} = 0.25$. Thus:

$$L = 8 - \frac{1}{4} = 7.75$$
$$c = y = 2L = 15.5$$

Output decreases as the consumer wants to work less which decreases the input L decreasing output.

Exercise 2

We need:

- Convexity of preferences

- Continuity of aggregate demand
- as $p_l \to 0$ we have that $z_l > 0$ and

  as $p_l \to \infty$ we have that $z_l < 0$

Suppose continuity is violated. As we see $z_l = 0$ does not occur. Without market clearing, there is no CE.

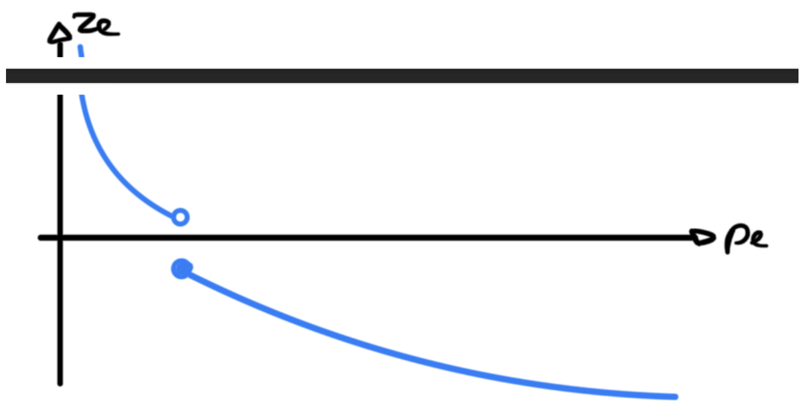


Figure 3: Ignore the horizontal line. It was created by accident when exporting the drawing from the iPad.

Exercise 3

$w^h = (2, 2)$

(a)

$$\text{at } t = 0 : q_1\theta_1^h + q_2\theta_2^h = 0$$
$$\text{at } t = 1 : x^h(1) = 2 + \theta_1^h$$
$$x^h(2) = 2 + \theta_2^h$$

Combine into one BC:

$$q_1\left[x^h(1)-2\right]+q_2\left[x^h(2)-2\right]=0$$

<u>Consumer h:</u>

$$\begin{aligned}&\max_{x^h(s)}\pi^h\ln\left(x^h(1)\right)+\left(1-\pi^h\right)\ln\left(x^h(2)\right)\\ &\text{s.t. }\; q_1\left[x^h(1)-2\right]+q_2\left[x^h(2)-2\right]=0\end{aligned}$$

FOCs

$$\begin{aligned}\pi^h\frac{1}{x^h(1)}-\lambda q_1&=0\\ (1-\pi^h)\frac{1}{x^h(2)}-\lambda q_2&=0\\ \implies\quad \frac{\pi^h}{1-\pi^h}x^h(2)&=\frac{q_1}{q_2}x^h(1)\end{aligned}$$

Plug into BC:

$$\begin{aligned}&\frac{\pi^h}{1-\pi^h}x^h(2)-2\frac{q_1}{q_2}+x^h(2)-2=0\\ &x^h(2)\left[\frac{\pi^h}{1-\pi^h}+1\right]=2\left(1+\frac{q_1}{q_2}\right)\\ &x^h(2)=2\left(1+\frac{q_1}{q_2}\right)\left(1-\pi^h\right)\end{aligned}$$

<u>Market Clearing:</u>

$$
\begin{aligned}
x^1(2) + x^2(2) &= 4 \\
2\left(1 + \frac{q_1}{q_2}\right)\left[1 - \pi^1 + 1 - \pi^2\right] &= 4 \qquad \text{(I)} \\
\overset{\pi^1=\pi^2}{\Longrightarrow} 4\left(1 + \frac{a_1}{q_2}\right)(1-\pi) &= 4 \\
\Longrightarrow \frac{q_1}{q_2} = \frac{1}{1-\pi} - 1 &= \frac{\pi}{1-\pi}
\end{aligned}
$$

Thus: $x^1(2) = x^1(1); x^2(1) = x^2(2)$ which I plug into BC:

$$
\begin{aligned}
(q_1 + q_2)\left[x^h(s) - 2\right] &= 0 \\
\iff \quad x^h(s) &= 2 \quad \forall s, h
\end{aligned}
$$

<u>Competitive Equilibrium:</u>

$$
\begin{aligned}
\left(x^1(1), x^1(2)\right) &= (2,2) \\
\left(x^2(1), x^2(2)\right) &= (2,2) \\
\frac{q_1}{q_2} &= \frac{\pi}{1-\pi}
\end{aligned}
$$

There is no trade. Reason being that the consumers are perfectly identical. There is no gain from exchanging anything.

(b) Everything up to (I) is identical. From there:

$$
\begin{aligned}
\left(1 + \frac{q_1}{q_2}\right)\left(2 - \pi^1 - \pi^2\right) &= 2 \\
\Longrightarrow \frac{q_1}{q_2} = \frac{2}{2 - \pi^1 - \pi^2} - 1 = \frac{\pi^1 + \pi^2}{(1-\pi^1) + (1-\pi^2)} &= 1
\end{aligned}
$$

Therefore we find

$$\begin{aligned}\big(x^1(1), x^1(2)\big) &= (1,3)\\ \big(x^2(1), x^2(2)\big) &= (3,1)\end{aligned}$$

Agent 1 increases (decreases) state 1 (2) consumption. Vice versa for agent 2. I.e. agent 1 buys asset 1 because she believes state 1 to be more likely. Thus it is optimal for her to insure against being poor in that state. Agent 2 does the opposite. Clearly there is trade through the Arrow securities.

4 Microeconomics Midterm 2014 / 15

Schmidt

Exercise 1

(a)

$$x_1(\lambda p, \lambda w) = \lambda^{1+\alpha-\delta} \frac{p_1^\alpha w}{p_1^\delta + p_2^\delta + p_3^\delta} = \lambda^{1+\alpha-\delta} x_1(p, w)$$

Must have $1 + \alpha - \delta = 0$ or $\alpha = \delta - 1$

$$x_2(\lambda p, \lambda w) = \lambda^{1+\alpha-\delta} \frac{p_2^\alpha w}{p_1^\delta + p_2^\delta + p_3^\delta} + \beta \frac{p_1}{p_3} \frac{\lambda}{\lambda}$$

No restriction on β.

$$x_3(\lambda p, \lambda w) = \lambda^{1+\alpha-\sigma} \frac{\gamma p_3^\alpha w}{p_1^\delta + p_2^\delta + p_3^\delta} = \lambda^{1+\alpha-\delta} x_3(p, w)$$

No restriction on γ.

In summary, we only need $\alpha = \delta - 1$.

(b)

$$\begin{aligned} p_1 x_1(\cdot) + p_2 x_2(\cdot) + p_3 x_3(\cdot) &= w \quad \text{to satisfy Walras' Law} \\ \Leftrightarrow \frac{w}{p_1^\delta + p_2^\sigma + p_3^\sigma} \left[p_1^{1+\alpha} + p_2^{1+\alpha} + \gamma p_3^{1+\alpha} \right] + \beta \frac{p_1 p_2}{p_3} &= w \end{aligned}$$

Must have $\beta = 0$:

$$p_1^\delta + p_2^\delta + p_3^\delta = p_1^{1+\alpha} + p_2^{1+\alpha} + \gamma p_3^{1+\alpha}$$

Must have $\gamma = 1$ & $\alpha = \delta - 1$.

In summary:

$$\alpha = \delta - 1 \quad \beta = 0 \quad \gamma = 1$$

Exercise 2

(a) Invert $e(p,u)$ as in equilibrium: $e(p,u) = w$ and also $u = v(p,w)$

$$v\left(p,w\right) = w\frac{p_1+p_2}{p_1p_2} = w\left[\frac{1}{p_1}+\frac{1}{p_2}\right]$$

(b) Roy's Identity

$$x_1\left(p_1w\right) = -\frac{\frac{\partial v(\cdot)}{\partial p_1}}{\frac{\partial v(\cdot)}{\partial w}} = \frac{w\frac{1}{p_1^2}}{\frac{p_1+p_2}{p_1p_2}} = \frac{w}{p_1+p_2}\frac{p_2}{p_1}$$

$$x_2\left(p_1w\right) = \frac{w}{p_1+p_2}\frac{p_1}{p_2} \text{ by symmetry}$$

(c)

$$\frac{x_1\left(p,w\right)}{x_2\left(p,w\right)} = \left(\frac{p_1}{p_2}\right)^{-2}$$

$$\eta_{12} = -(-2)\left(\frac{p_1}{p_2}\right)^{-3}\frac{\frac{p_1}{p_2}}{\left(\frac{p_1}{p_2}\right)^{-2}} = 2$$

Exercise 2

(a) Invert $e(p,u)$ as in equilibrium: $e(p,u) = w$ and also $u = v(p,w)$

$$v(p,w) = w\frac{p_1+p_2}{p_1p_2} = w\left[\frac{1}{p_1}+\frac{1}{p_2}\right]$$

(b) Roy's Identity

$$x_1(p_1w) = -\frac{\frac{\partial v(\cdot)}{\partial p_1}}{\frac{\partial v(\cdot)}{\partial w}} = \frac{w\frac{1}{p_1^2}}{\frac{p_1+p_2}{p_1p_2}} = \frac{w}{p_1+p_2}\frac{p_2}{p_1}$$
$$x_2(p_1w) = \frac{w}{p_1+p_2}\frac{p_1}{p_2} \text{ by symmetry}$$

(c) CES utility:

$$u(x_1,x_2) = \left[\frac{1}{2}x_1^\rho + \frac{1}{2}x_2^\rho\right]^{\frac{1}{\rho}} \text{ where } \rho = 1-\frac{1}{n_{12}} = 1/2$$

Exercise 3

The difference between consumer theory and production theory is mainly the fact that firms do not have budget constraints. This problem introduces a budget constraint. Therefore, we are going to treat the problem like a consumer problem. In that sense, the revenue is comparable to the utility function, and the cash constraint is like the wealth of a consumer. Consequently, we are solving the following revenue maximization problem (which is the analogue to a utility maximization problem):

$$\max_{z_1,z_2} pf(z_1,z_2)$$
$$\text{s.t. } w_1z_1+w_2z_2 \le C$$

We will assume an interior solution (the budget constraint is binding). Then, the revenue function $R(p,w_1,w_2,C)$ that the exercise gives us is just the equivalent to the indirect utility.

(a) As $R(p, w_1, w_2, C)$ works like the indirect utility, we apply Roy's identity to find the factor demand, which is the analogue to the Walrasian demand:

$$
\begin{aligned}
z_1 &= -\frac{\frac{\partial R}{\partial w_1}}{\frac{\partial R}{\partial C}} \\
&= -\frac{p \cdot (-\alpha)\frac{1}{w_1}}{p \cdot \frac{1}{C}} \\
&= \alpha \frac{C}{w_1}
\end{aligned}
$$

(b) We treat $R(p, w, C)$ as the indirect utility depending on income and invert it to find the cost function $C(p, w, R)$, which is the analogue to the expenditure function in consumer theory:

$$
\begin{aligned}
R &= p\left[\gamma + \ln C(p, w, R) - \alpha \ln w_1 - (1-\alpha)\ln w_2\right] \\
\frac{R}{p} - \gamma &= \ln\left(\frac{C(p, w, R)}{w_1^{\alpha} w_2^{1-\alpha}}\right) \\
\exp\left(\frac{R}{p} - \gamma\right) &= \frac{C(p, w, R)}{w_1^{\alpha} w_2^{1-\alpha}} \\
C(p, w, R) &= w_1^{\alpha} w_2^{1-\alpha} \exp\left(\frac{R}{p} - \gamma\right)
\end{aligned}
$$

(c) Since the cost function from (b) happens to be the analogue to the expenditure function, we can apply Shephard's Lemma in order to find the factor demand for a given R at minimum cost, as this is the analogue to the Hicksian demand in consumer theory. In that spirit, let us call this function $h_1(p, w, R)$.

$$\begin{aligned}
h_1(p, w, R) &= \frac{\partial C\left(w, R\right)}{\partial w_1} \\
&= \alpha \exp\left[\frac{R}{p} - \gamma\right] \cdot \left(\frac{w_2}{w_1}\right)^{1-\alpha}
\end{aligned}$$

(d) In consumer theory, the Hicksian demand and the Walrasian demand meet at optimum. We can also show that here:

$$\begin{aligned}
h_1(w, R) &= z_1^* \\
\alpha \exp\left[\frac{R}{p} - \gamma\right] \cdot \left(\frac{w_2}{w_1}\right)^{1-\alpha} &= \alpha \frac{C}{w_1} \\
\exp\left[\frac{R}{p} - \gamma\right] w_1^{\alpha} w_2^{1-\alpha} &= C \\
\frac{R}{p} - \gamma &= \ln\left(\frac{C}{w_1^{\alpha} w_2^{1-\alpha}}\right) \\
R &= p\left[\gamma + \ln C - \alpha \ln w_1 - (1-\alpha)\ln w_2\right]
\end{aligned}$$

The last line is exactly the formula for the revenue that is observed by our econometrician friend in the optimum. Therefore, we have shown that the two demands are equal whenever the firm is acting optimally, i.e. maximizing its revenue or minimizing its cost. Put differently, the revenue maximization problem is the dual problem to the cost minimization problem and vice versa.

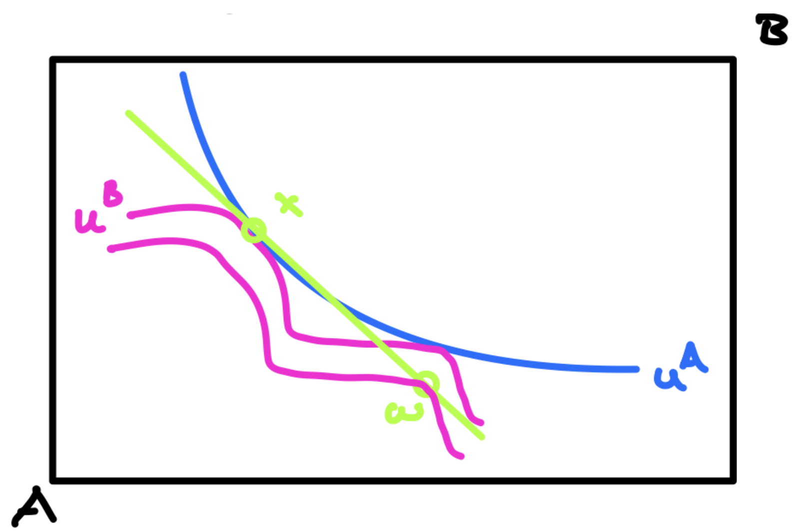
Gottardi

Exercise 1

(a) True. We need three things for FWT:

- LNS, which is satisfied by monotonicity
- Complete markets, satisfied by two prices for two commodities
- free disposal (given)

(b) False. Convexity is violated by B. Consider the following illustration:



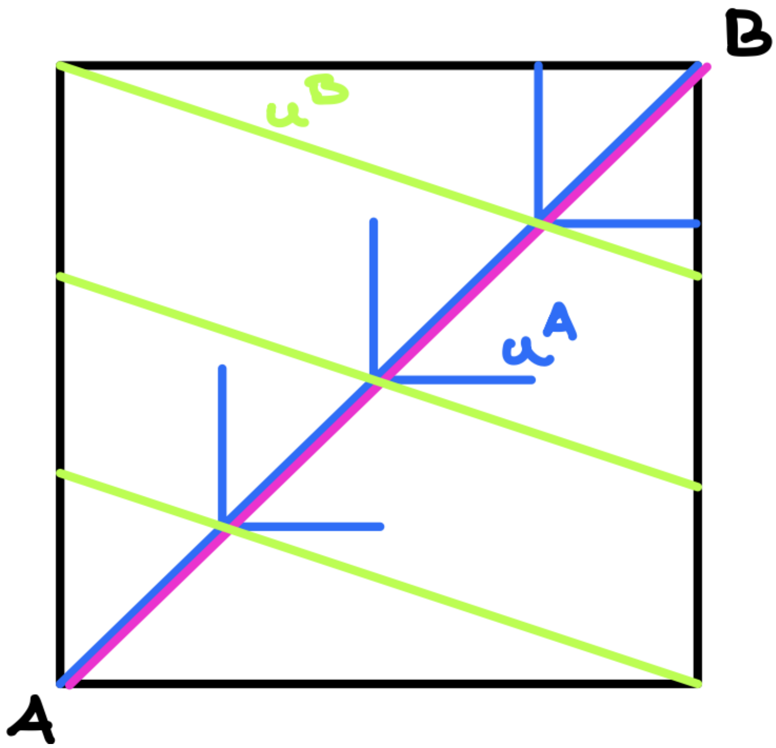
Because B has non-convex preferences, x is not a CE. Actually no CE exists.

(c) False by same argument as in (b).

Exercise 2

(a) PE allocations are along $x_1^A = x_2^A$. If we are at any other point, just give some to B because A only cares about lower amount.

(b) Let $p = \frac{p_1}{p_2}$



Consumer A:

$$x_1^A = x_2^A \quad \text{BC: } px_1^A + x_2^A = 6p + 2$$

Consumer B:

$$x_1^B = \begin{cases} \infty & \text{if} \quad p < 1/3 \\ \mathbb{R}^+ & \text{if} \quad p = 1/3 \\ 0 & \text{if} \quad \rho > 1/3 \end{cases} \quad ; \quad x_2^B = \begin{cases} \infty & \text{if} \quad p > 1/3 \\ \mathbb{R}^+ & \text{if} \quad p = 1/3 \\ 0 & \text{if} \quad p < 1/3 \end{cases}$$

$$\text{BC: } px_1^B + x_2^B = 2p + 6$$

Market Clearing:

$$x_1^A + x_1^B = w_1^A + w_1^B = 8$$
$$x_2^A + x_2^B = w_2^A + w_2^B = 8$$

use $x_1^A = x_2^A \longrightarrow x_1^B = x_2^B$. Therefore $p = \frac{1}{3}$ so no excess demand for either good.

By BC^A:

$$x_1^A = x_2^A = 3$$
$$x_1^B = x_2^B = 5$$

<u>Competitive Equilibrium:</u>

$$\left(x_1^A, x_2^A\right) = (3, 3)$$
$$\left(x_1^B, x_2^B\right) = (5, 5)$$
$$p = \frac{1}{3}$$

This is PE since $x_1^A = x_2^A$.

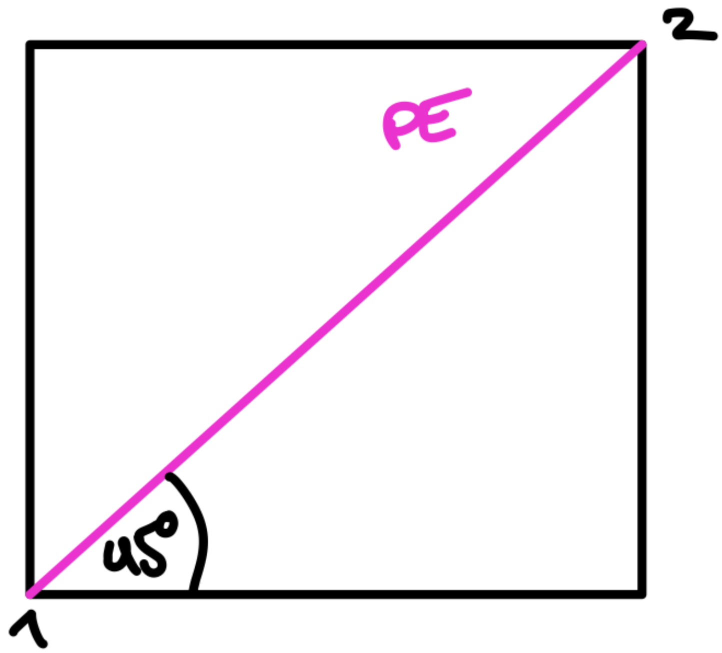
(c) Yes. The reason is that $p = \frac{1}{3}$ is the only possible equilibrium price. Otherwise markets cannot clear & we have excess demand for one of the commodities.

Exercise 3

$w^1 = (8, 4)$; $\quad w^2 = (2, 6)$

(a) Note that we have (1) identical beliefs and (2) no aggregate risk as $w_1 = w_2 = 10$.

Therefore, full risk sharing is possible and $x_1^h = x_2^h \quad \forall h$ is PE. I.e. the 45°-line:



(b) <u>Consumer h:</u>

$$\begin{aligned}
\max_{x_1^h, x_2^h} \quad & \pi u^h\left(x_1^h\right) + (1-\pi) u^h\left(x_2^h\right) \\
\text{s.t.} \quad & q_1 \theta_1^h + q_2 \theta_2^h = 0 \\
& x_1^n = w_1^h + \theta_1^h \\
& x_2^u = w_1^h + \theta_2^h
\end{aligned}$$

Plug in the θs:

$$
\begin{aligned}
\max_{x_1^h, x_2^h} \quad & \pi u^h\left(x_1^h\right) + (1-\pi) u^h\left(x_2^h\right) \\
\text{s.t.} \quad & q_1\left(x_1^h - w_1^h\right) + q_2\left(x_2^h - w_2^h\right) = 0
\end{aligned}
$$

FOCs:

$$
\begin{aligned}
\pi \frac{\partial u^h\left(x_1^h\right)}{\partial x_1^h} - \lambda q_1 &= 0 \\
(1-\pi) \frac{\partial u^h\left(x_2^h\right)}{\partial x_2^h} - \lambda q_2 &= 0 \\
\implies \quad \frac{q_1}{q_2} &= \frac{\pi}{1-\pi} \frac{\frac{\partial u^h\left(x_1^h\right)}{\partial x_1^h}}{\frac{\partial u^h\left(x_2^h\right)}{\partial x_2^h}}
\end{aligned}
$$

Perfect risk sharing implies: $x_1^h = x_2^h$, and therefore we have

$$
\frac{q_1}{q_2} = \frac{\pi}{1-\pi}
$$

Plug this into the BC:

$$
\begin{aligned}
\frac{q_1}{q_2}\left(x_1^h - w_1^h\right) + x_1^h - w_2^h &= 0 \\
x_1^h\left(\frac{q_1}{q_2} + 1\right) &= w_2^h + w_1^h \frac{q_1}{q_2} \\
x_1^h = x_2^h = \left(\frac{q_1}{q_2} + 1\right)^{-1}\left(w_2^h + w_1^h \frac{q_1}{q_2}\right) &= (1-\pi)\left(w_2^h + w_1^h \frac{\pi}{1-\pi}\right) = \pi w_1^h + (1-\pi) w_2^h \\
x_1^1 = x_2^1 &= \pi 8 + (1-\pi) 4 = 4(1+\pi) \\
x_1^2 = x_2^2 &= \pi 2 + (1-\pi) 6 = 6 - 4\pi
\end{aligned}
$$

Competitive Equilibrium:

$$\begin{aligned}
\left(x_1^1, x_2^1\right) &= (4 + 4\pi, 4 + 4\pi) \\
\left(x_1^2, x_2^2\right) &= (6 - 4\pi, 6 - 4\pi) \\
\frac{q_1}{q_2} &= \frac{\pi}{1 - \pi}
\end{aligned}$$

5 Microeconomics Midterm 2015 / 16

Schmidt

Exercise 1

(a) To violate WARP:

find w by Walras law

$$\left| \begin{array}{l} p'y \leqslant w' \\ py' \leqslant w \end{array} \right|$$
$$\Leftrightarrow \left| \begin{array}{l} 30(12+x) \leqslant 600 \\ 10(30+24) \leqslant 360+24x \end{array} \right|$$
$$\Leftrightarrow \left| \begin{array}{l} 30x \leqslant 240 \\ 180 \leqslant 24x \end{array} \right|$$
$$\Leftrightarrow \left| \begin{array}{l} 30x \leqslant 240 \\ 180 \leqslant 24x \end{array} \right|$$
$$\Leftrightarrow \left| \begin{array}{c} x \leqslant 8 \\ 7.5 \leqslant x \end{array} \right|$$

WARP is violated for $x \in [7.5, 8]$.

(b) Bundle 2 must be affordable in period 1. Then we see that the consumer chooses bundle 1 over bundle 2: $x \leqslant 8$ by (a)

To not violate WARP we find that if and only if $x \in [0, 7.5)$, bundle 1 is revealed preferred.

(c) The quantity increased. To have an inferior good, $\frac{\partial y_1}{\partial w} < 0$. Thus, income must have decreased:

$$600 \leqslant 360 + 24x$$
$$\Leftrightarrow 10 \leqslant x$$

We find that good 1 is inferior for $x \geqslant 10$.

Exercise 2

Monotone transformation to $u_2(\cdot)$:

$$\tilde{u}_2(x_1, x_2) = x_1^{\frac{3}{3+a}} x_2^{\frac{a}{3+a}}$$

(a) consumer 1: Invert $e_1(\cdot)$. In equilibrium $e_1(\cdot) = w_1$ and $v_1(p, w_1) = u_1$:

$$\begin{aligned} & w_1 = v_1(p, w_1)\sqrt{p_1 p_2} \\ \Leftrightarrow \quad & v_1(p, w_1) = \frac{w_1}{\sqrt{p_1 p_2}} \end{aligned}$$

Apply Roy's identity:

$$x_1^1(p_1, w_1) = -\frac{\frac{\partial v_1(\cdot)}{\partial p_1}}{\frac{\partial v_1(\cdot)}{\partial w}} = -\frac{-\frac{1}{2}p_1^{-3/2}\frac{w}{\sqrt{p_2}}}{\frac{1}{\sqrt{p_1 p_2}}}$$

$$x_1^1(p_1, w_1) = \frac{1}{2}\frac{w_1}{p_1}$$

By symmetry of $v_1(p, w_1)$:

$$x_2^1(p_1, w_1) = \frac{1}{2}\frac{w_1}{p_2}$$

consumer 2: As $\tilde{u}_2(\cdot)$ is standard Cobb-Douglas, the result is immediate:

$$\begin{aligned} x_1^2(p, w_2) &= \frac{3}{3+a}\frac{w_2}{p_1} \\ x_1^2(p, w_2) &= \frac{a}{3+a}\frac{w_2}{p_2} \end{aligned}$$

(b) Aggregate demand: $x_l = x_l^1 + x_l^2$

Good 1: $x_1 = \frac{1}{p_1}\left[\frac{1}{2}w_1 + \frac{3}{3+a}w_2\right] \rightarrow \frac{1}{2} \overset{!}{=} \frac{3}{3+a}$

Good 2: $x_2 = \frac{1}{p_2}\left[\frac{1}{2}w_1 + \frac{a}{3+a}w_2\right] \rightarrow \frac{1}{2} \overset{!}{=} \frac{a}{3+a}$

In both cases: $a = 3$

Exercise 3

(a) $R = -CV$ as it is the amount that has to be given after implementing the change.

The Leontief preferences imply that they must be able to afford the old bundle & they will buy it.

By Leontief: $x_1 = x_2 \rightarrow w = (p_1 + p_2)\, x_1$

Before moving: $1000 = 2 \cdot x_1 \Leftrightarrow x_1 = x_2 = 500$

After moving: $1000 + R = 5 \cdot x_1 \Leftrightarrow R = 5x_1 - 1000$

As discussed, must choose same bundle to have $u_0 = u_1 = 500$.

$$R = 2500 - 1000 = 1500$$

(b) Cobb-Daglas implies: $x_l = \frac{1}{2}\frac{w}{p_l}$

Before moving:

$$x_1 = x_2 = 500 \rightarrow u_0 = 500$$

After moving:

$$\begin{aligned} x_1 &= \frac{1}{2}\frac{(1000 + R)}{4} \\ x_2 &= \frac{1}{2}(1000 + R) \\ u_1 &= \frac{1000 + R}{2}\frac{1}{2} \overset{!}{=} u_0 = 500 \\ &\Leftrightarrow R = 1000 \end{aligned}$$

We plug R into demands: $x_1 = 250$; $x_2 = 1000$.

The demand for x_1 decreased and it increased for x_2. Reason being that Cobb-Douglas (unlike Leontief) allows for substitution. Therefore, demand followed the price change.

Gottardi

Exercise 1

Only relative prices matter. Define $p^{\text{aut}} = \frac{p_1^{\text{aut}}}{p_2^{\text{aut}}}$ and $p = \frac{p_1}{p_2}$.

Case 1: ($p = p^{\text{aut}}$)

Nothing changes. No welfare effects.

Case 2: ($p > p^{\text{aut}}$)

Assume A sells commodity 1. Then the price increase benefits her, as she can sell at a higher price.

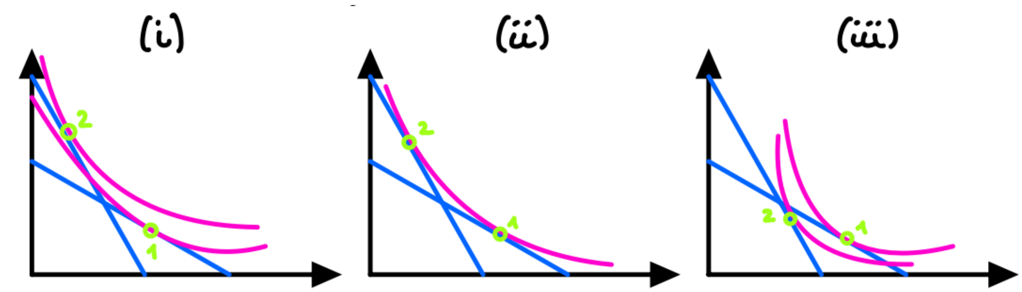
Assume A buys commodity 1. The effect depends on her ability to substitute, which depends on her preferences. There three three options:

(i) She switches to selling commodity 1. The price change is beneficial.

(ii) She can substitute without gaining from it in terms of utility.

(iii) She cannot substitute sufficiently and the price change hurts her.

The graphs illustrate the three cases. Green are equilibria. 1 is under autarky & 2 after opening p. Blue are budget sets & pink are indifference curves.



Case 3: ($p < p^{\text{aut}}$)

Substitute A sells 1 and buys 1 in case 2. The argument is just the inverse.

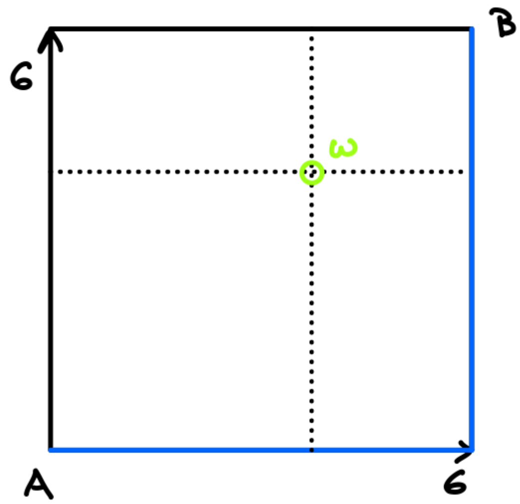
For agent B the argument is always just the inverse in all cases.

We see that at least one agent is weakly better off.

Exercise 2

(a) Since A cares more about x_1 and B more about x_2, the PE allocations ce around the edges of the box (in blue).

$$PE = \left\{x_2^A = 0 \text{ or } x_1^B = 0\right\}$$



(b) For PE, equate MRS across agents:

$$\text{MRS}^A = 2 \overset{!}{=} \text{MRS}^B = \frac{x_2^B}{x_1^B} \Leftrightarrow x_2^B = 2x_1^B$$

For CE consumers behave optimally & markets must clear. Let $p = p_1/p_2$

A: By linearity of preferences:

$$x_1^A = \begin{cases} \infty & \text{if } p < 2 \\ \mathbb{R}^+ & \text{if } p = 2 \\ 0 & \text{if } p > 2 \end{cases} \quad x_2^A = \begin{cases} \infty & \text{if } p > 2 \\ \mathbb{R}^+ & \text{if } p = 2 \\ 0 & \text{if } p < 2 \end{cases}$$

B:

$$\max_{C_1^8, c_2^8} \ln\left(x_1^8\right) + \ln\left(x_2^8\right)$$
$$\text{s.t. } px_1^B + x_2^B = p^2 + 2$$

FOCs

$$\frac{1}{x_1^B} - \lambda p = 0$$
$$\frac{1}{x_2^B} - \lambda = 0$$
$$\implies x_2^B = x_1^B p$$

Plug into BC :

$$x_1^B = \frac{p+1}{p}; x_2^B = p+1$$

<u>Market Clearing:</u>

For markets to clear we have $p = 2$. Otherwise A will have infinite demand for one of the goods:

$$x_1^B = \frac{3}{2}; \quad x_2^B = 3$$
$$x_1^A = 6 - x_1^3 = \frac{9}{2}; \quad x_2^A = 6 - x_2^B = 3$$

<u>Competitive Equilibrium:</u>

$$\left(x_1^A, x_2^A\right) = (\frac{9}{2}, 3)$$
$$\left(x_1^B, x_2^B\right) = (\frac{3}{2}, 3)$$
$$p = 2$$

It is PE since $x_2^B = 3 = px_1^B = 2\frac{3}{2}$.

Exercise 3

$$w^A = (8, 4); w^B = (2, 4)$$

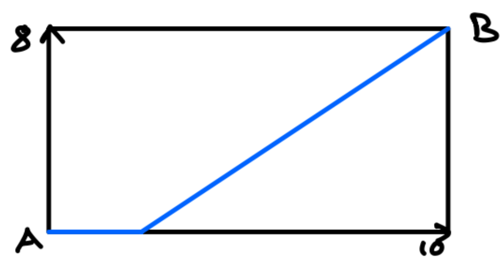
(a)

$$MRS^A = \frac{1/2}{1/2} = 1 \stackrel{!}{=} MRS^B = \frac{1/2}{1/2}\frac{\frac{\partial u^B(\cdot)}{\partial x_1}}{\frac{\partial u^B(\cdot)}{\partial x_2}}$$

$$\iff \frac{\partial u^B(\cdot)}{\partial x_1} = \frac{\partial u^B(\cdot)}{\partial x_2} \iff x_1^B = x_2^B$$

PE in blue. Defined by

$$x_2^B = \left\{ \begin{array}{ll} x_1^B & \text{if } x_1^B \leq 8 \\ 8 & \text{else} \end{array} \right\}$$



(b) Budget constraints:

$$\begin{aligned} \text{at } t = 0: \quad & q_1\theta_1^h + q_2\theta_2^h = 0 \\ \text{at } t = 1: \quad & x_1^h = w_1^h + \theta_1^h \\ & x_2^h = w_2^h + \theta_2^h \end{aligned}$$

Plug the θs into first BC:

$$q_1\left(x_1^h - w_1^h\right) + q_2\left(x_2^h - w_2^h\right) = 0$$

UMP:

$$\max_{x_1^h, x_2^h} \quad \pi_1 u^h\left(x_1^h\right) + \pi_2 u^h\left(x_2^h\right)$$
$$\text{s.t. } q_1\left(x_1^h - w_1^h\right) + q_2\left(x_2^h - w_2^h\right) = 0$$

FOCs:

$$\pi_1 \frac{\partial u^h(\cdot)}{\partial x_1^h} - \lambda q_1 = 0$$
$$\pi_2 \frac{\partial u^h(\cdot)}{\partial x_2^h} - \lambda q_2 = 0$$
$$\longrightarrow \frac{q_1}{q_2} = \frac{\pi_1}{\pi_2} \frac{\partial u^h(\cdot)}{\partial x_1^h} \left(\frac{\partial u^h(\cdot)}{\partial x_2^h}\right)^{-1} \tag{I}$$

<u>Agent A:</u>

By $\pi_1 = \pi_2$ & linearity:

$$\frac{q_1}{q_2} = 1$$

<u>Agent B:</u>

Plug $\frac{q_1}{q_2} = 1$ into (I) and also $\pi_1 = \pi_2$:

$$\frac{\partial u^B(\cdot)}{\partial x_1^B} = \frac{\partial u^B(\cdot)}{\partial x_2^B} \Leftrightarrow x_1^B = x_2^B \tag{II}$$

Plug (II) into BC of B using $\frac{q_1}{q_2} = 1$:

$$x_1^B = x_2^B = \frac{w_1^B + w_2^B}{2} = 3$$

<u>Market Clearing:</u>

$$x_1^A \overset{!}{=} w_1^A + w_1^B - x_1^B = 7$$
$$x_2^A \overset{!}{=} w_2^A + w_2^B - x_2^B = 5$$

Competitive Equilibrium:

$$\left(x_1^A, x_2^A\right) = (7, 5)$$
$$\left(x_1^B, x_2^B\right) = (3, 3)$$
$$\frac{q_1}{q_2} = 1$$

It is PE as $x_1^B < 8$ and $x_2^B = x_1^B$.

(c) By (I) we know that $\frac{q_1}{q_2} > 1$. Thus insurance for B is more expensive & she buys less of it. At the same time, A believes $s = 1$ to be more likely. Therefore, A consumes more in $s = 1$, and B less. A consumes less in $s = 2, B$ consumes more.

6 Microeconomics Midterm 2016 / 17 (1)

There are somehow two midterms for this year. This is one of the exams.

Schmidt

Exercise 1

(a) Use Roy's identity on some monotonic transformation $f(\cdot)$ of $v(p,w)$:

$$\begin{aligned}\tilde{x}_l(p,w) &= -\frac{\frac{\partial f(v(p,w))}{\partial p_l}}{\frac{\partial f(v(p)w)}{\partial w}} = -\frac{\frac{\partial f(v(p,w))}{\partial v(p,w)}\cdot\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial f(p(p,w))}{\partial v(p,w)}\cdot\frac{\partial v(p,w)}{\partial w}} \\ &= -\frac{\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial v(p,w)}{\partial w}} = x_l(p,w)\end{aligned}$$

(b) (1) invert $v(p,w)$ to obtain e(p,u). In equilibrium

$$\begin{aligned} v\left(p_1,w\right) = u \quad &; \quad e\left(p_1,u\right) = w \\ u = \left(\frac{\alpha}{p_1}\right)^{\alpha}&\left(\frac{1-\alpha}{p_2}\right)^{1-\alpha} e\left(p_1,u\right) \\ \Leftrightarrow \quad e\left(p_1,u\right) &= u\left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha}\end{aligned}$$

(2) Apply Shepherd's Lemma:

$$\begin{aligned} h_1\left(p_1,u\right) = \frac{\partial e\left(p_1 u\right)}{\partial p_1} &= u\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} k p_1^{\alpha-1} \\ &= u\left(\frac{p_2\alpha}{(1-\alpha)p_1}\right)^{1-\alpha}\end{aligned}$$

(c) <u>case 1:</u>

$$
\begin{aligned}
\alpha &= \alpha\left(p_1/p_2\right) \longrightarrow \alpha\left(\lambda p_1/\lambda p_2\right) = \alpha\left(p_1/p_2\right) \\
h_1\left(\lambda p_1, u\right) &= u\left(\frac{\lambda p_2 \alpha\left(p_1/p_2\right)}{\left(1-\alpha\left(p_1/p_2\right)\right)\lambda p_1}\right)^{1-\alpha\left(p_1/p_2\right)} \\
&= u\left(\frac{p_2 \alpha\left(p_1/p_2\right)}{\left(1-\alpha\left(p_1/p_2\right)\right) p_1}\right)^{1-\alpha\left(p_1/p_2\right)} = h_1\left(p_1, u\right)
\end{aligned}
$$

case 2: $\alpha = \alpha\left(p_1\right) \longrightarrow \alpha\left(\lambda p_1\right) \neq \alpha\left(p_1\right)$

$$
\begin{aligned}
h_1\left(\lambda p_1, u\right) &= u\left(\frac{\lambda p_2 \alpha\left(\lambda p_1\right)}{\left(1-\alpha\left(\lambda p_1\right)\right)\lambda p_1}\right)^{1-\alpha\left(\lambda p_1\right)} \\
&= u\left(\frac{p_2 \alpha\left(\lambda p_1\right)}{\left(1-\alpha\left(\lambda p_1\right)\right) p_1}\right)^{1-\alpha\left(\lambda p_1\right)} \neq h_1\left(p_1, u\right)
\end{aligned}
$$

Exercise 2

$$A = -EV \quad ; \quad B = -CV$$

Note that we can transform $U\left(x_1, x_2\right)$ to have Cobb-Douglas:

$$\tilde{u}\left(x_1, x_2\right) = \left(x_1 x_2\right)^{1/2}$$

Thus:

$$
\begin{aligned}
x_1(p, w) &= \frac{1}{2}\frac{w}{p_1}; x_2(p, w) = \frac{1}{2}\frac{w}{p} \\
v(p, w) &= \frac{w}{2}\left(\frac{1}{p_1 p_2}\right)^{1/2}
\end{aligned}
$$

Before moving:

$$v_0(p, w) = \frac{3000}{2} = 1500$$

After moving (no raise):

$$v_1(p, w) = \frac{3000}{2}\left(\frac{1}{2.25}\right)^{1/2} = 1000$$

Calculate A by subtracting from w and let $p_1 = p_2 = 1$.
Same utility as after move without raise:

$$\begin{aligned} v_1(p,w) &= \frac{w-A}{2} \\ 1000 &= \frac{3000-A}{2} \\ A &= 1000 \end{aligned}$$

Calculate B by adding to w letting $p_1 = 1, p_2 = 2.25$.
same utility as before moving:

$$\begin{aligned} v_0(p,w) &= \frac{w+B}{2}\left(\frac{1}{p_1 p_2}\right)^{1/2} \\ 1500 &= \frac{3000+B}{2}\left(\frac{1}{2.25}\right)^{1/2} \\ B &= 1500 \end{aligned}$$

Exercise 3

(a) (1) Solve CMP for $f(x) = 1$:

$$\begin{aligned} &\min_x wx \\ \text{s.t. } &f(x) = 1 \end{aligned}$$

First order conditions:

$$
\begin{aligned}
w_l - \lambda \frac{\partial f(x)}{\partial x_l} &= 0 \quad \forall l \mid \cdot x_l \\
w_l x_l - \lambda \frac{\partial f(x)}{\partial x_l} x_l &= 0 \quad \forall l \mid \text{ sum over } l \\
\sum_l w_l x_l - \lambda \sum_l \frac{\partial f(x)}{\partial x_l} x_l &= 0 \\
wx &= \lambda \sum_l \frac{f(x)}{\partial x_l} x_l \mid \text{ by CRS apply Euler} \\
wx &= \lambda f(x) \mid \text{ use } f(x) = 1 \\
wx &= \lambda = c(w)
\end{aligned}
$$

(2) Solve CMP for $f(x) = y$:

$$
\begin{aligned}
&\min_x wx \\
&\text{s.t. } f(x) = y
\end{aligned}
$$

Up to $wx = \lambda f(x)$ everything is identical:

$$
\begin{aligned}
wx &= \lambda f(x) \quad \mid \text{ use } f(x) = y \\
wx &= \lambda y = c(w, y) = c(w) \cdot y
\end{aligned}
$$

(b) Apply Shephard's Lemma:

$$
\begin{aligned}
x_l(w) &= \frac{\partial c(w)}{\partial w_l} \\
x_l(w, y) &= \frac{\partial c(w, y)}{\partial w_l} = y \frac{\partial c(w)}{\partial w_l} = y \cdot x_l(w)
\end{aligned}
$$

(c) Profits are

$$
\begin{aligned}
\pi &= pf(x) - wx \\
&= pf(x) - \sum_l p \frac{\partial f(x)}{\partial x_l} x_l \\
&= p \left[f(x) - \sum_l \frac{\partial f(x)}{\partial x_l} x_l \right]
\end{aligned}
$$

By Euler & CRS we know that

$$
\sum_l \frac{\partial f(x)}{\partial x_l} x_l = f(x)
$$

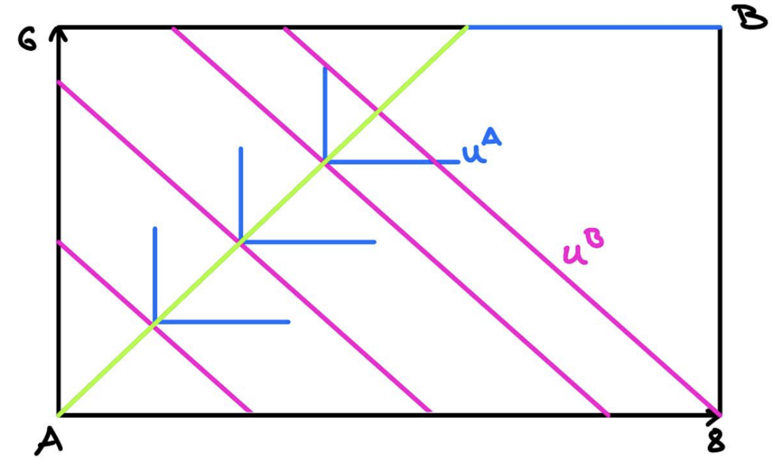
Thus :

$$
\pi = p \left[f(x) - \sum_l \frac{\partial f(x)}{\partial x_l} x_l \right] = 0
$$

Gottardi

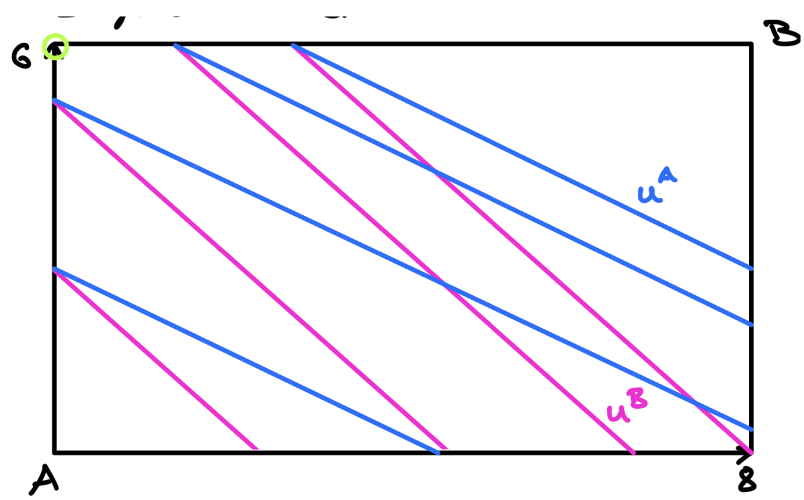
Exercise 1

(a) For A must have $x_1^A = x_2^A$ while B is happy with having only one good. Easiest to look at it in Edgeworth box:



Look at ind. curves to see that only PE allocations are on $x_1^A = x_2^A$ as long os $x_1^A < 6$ Green or PE.

If $u^A\left(x_1^A, x_2^A\right) = x_1^A + 2x_2^A$ wed have the following Edgeworth Box:



Only PE allocation is the top left corner. As A values x_2 more, she gets all of it.

(b) Let $p = p_1/p_2$.

consumer A: Leontief implies $x_1^A = x_2^A$

consumer B : This is Cobb-Douglas with $\alpha = 1/2$. Thus:

$$x_2^8 = \frac{6}{2} = 3 \text{ and } x_1^B = \frac{6}{2p} = \frac{3}{p}$$

markets: $x_1^A + x_1^B = 6 \Leftrightarrow x_1^A = 3$

Use this in $x_1^A = x_2^A$: $x_2^A = 3$

$$x_2^A + x_2^B = 8 \Longleftrightarrow x_2^B = 5$$

Determine price:

$$x_2^B = 3/p \Leftrightarrow p = 3/5$$

<u>Competitive Equilibrium:</u>

$$\begin{aligned} \left(x_1^A, x_2^A\right) &= (3,3) \\ \left(x_1^B, x_2^3\right) &= (3,5) \\ p &= 3/5 \end{aligned}$$

(c) Look at excess demand:

$$\begin{aligned} z_1 &= x_1^A + x_1^B - w_1^A = x_1^A + \frac{3}{p_1} - 8 \\ \frac{\partial z_1}{\partial p_1} &= -\frac{3}{p_1^2} < 0 \\ z_2 &= x_2^A + x_2^B - w_2^B = x_2^A + \frac{3}{p_2} - 6 \\ \frac{\partial z_2}{\partial p_2} &= -\frac{3}{p_2^2} < 0 \end{aligned}$$

Excess demand is upward sloping, thus the CE is unique.

It is PE by FWT. We have LNS of preferences, complete markets, and free disposal.

Exercise 2

$$w^A = (12, 2) \quad w^B = (2, 8)$$

(a) for either consumer $h \in \{A, B\}$

at $t = 0$:

$$q_1 \theta_1^h + q_2 \theta_2^h = 0$$

at $t = 1$:

$$\begin{aligned} x_1^h &= \theta_1^h + w_1^h \\ x_2^h &= \theta_2^h + w_2^h \end{aligned}$$

Together:

$$q_1 \left(x_1^h - w_1^h\right) + q_2 \left(x_2^h - w_2^h\right) = 0$$

(b) <u>consumer h:</u>

$$\begin{aligned} &\max_{x_1^h, x_2^h} \pi_1^h u^h \left(x_1^h\right) + \pi_2^h u^h \left(x_2^h\right) \\ \text{s.t. } & q_1 \left(x_1^h - w_1^h\right) + q_2 \left(x_2^h - w_2^h\right) = 0 \end{aligned}$$

FOCs:

$$\begin{aligned} \left[x_1^h\right] : \pi_1^h \frac{\partial u^h \left(x_1^h\right)}{\partial x_1^h} - \lambda q_1 = 0 \\ \left[x_2^h\right] : \pi_2^h \frac{\partial u^h \left(x_2^h\right)}{\partial x_2^h} - \lambda q_2 = 0 \end{aligned}$$

Note that by risk-neutrality of A, we have $\frac{\partial u^A(.)}{\partial x_2^A} = 1$:

$$\frac{q_1}{q_2} = \frac{\pi_1^A}{\pi_2^A} = 1$$

If we plug this into the FOCs for B, we obtain:

$$\begin{aligned}
1 = \frac{q_1}{q_2} &= \frac{\pi_1^B \frac{\partial u^B\left(x_1^B\right)}{\partial x_1^B}}{\pi_2^B \frac{\partial u^B(x^B)}{\partial x_2^B}} = \frac{\frac{\partial u^B\left(x_1^B\right)}{\partial x_1^B}}{\frac{\partial u^B(x^B)}{\partial x_2^B}} \\
\Leftrightarrow \frac{\partial u\left(x_1^B\right)}{\partial x_1^B} &= \frac{\partial u\left(x^B\right)}{\partial x_2^B} \\
\Leftrightarrow x_1^B &= x_2^B
\end{aligned}$$

Plug this into BC for B :

$$\begin{aligned}
\frac{q_1}{q_2}\left(x_1^B - w_1^B\right) + \left(x_2^B - w_2^B\right) &= 0 \\
x_1^B - 2 + x_1^B - 8 &= 0 \\
x_1^B &= 5 = x_2^B
\end{aligned}$$

<u>markets:</u>

$$\begin{aligned}
x_1^A + x_1^B &= 14 \Rightarrow x_1^A = 9 \\
x_2^A + x_2^B &= 10 \Rightarrow x_2^A = 5
\end{aligned}$$

<u>Competitive Equilibrium:</u>

$$\begin{aligned}
\left(x_1^A, x_2^A\right) &= (9,5) \qquad \frac{q_1}{q_2} = 1 \\
\left(x_1^B, x_2^B\right) &= (5,5)
\end{aligned}$$

Due to risk neutrality of A, she carries all the risk while B perfectly smooths her consumption. At the same time this implies that A's beliefs determine the price ratio of the securities. By $\pi_1^A = \pi_2^A$ we have $q_1 = q_2$.

(c) No. The prices will change as they reflect A's beliefs:

$$\frac{q_1}{q_2} = \frac{\pi_1^A}{\pi_2^A} > 1$$

Use B's budget constraint: (sill $x_1^B = x_2^B$)

$$\begin{aligned}
&q_1\left(x_1^B - 2\right) + q_2\left(x_1^B - 8\right) = 0 \\
&x_1^B\left(q_1 + q_2\right) = 2q_1 + 8q_2 \\
&x_1^B = \frac{2q_1 + 8q_2}{q_1 + q_2} = \frac{2\frac{q_1}{q_2} + 8}{1 + q_1/q_2} \\
&\frac{\partial x_1^B}{\partial q_1/q_2} = \frac{2\left(1 + q_1/q_2\right) - \left(2q_1/q_2 + 8\right)}{\left(1 + q_1/q_2\right)^2} = \frac{-6}{\left(1 + q_1/q_2\right)^2} < 0
\end{aligned}$$

We see that B consumes less which makes sense as the asset that would insure her against her poor state ($s = 1$) has become more expensive & she purchases less of it.

By market clearing, A consumes more.

7 Microeconomics Midterm 2016 / 17 (2)

There are somehow two midterms for this year. This is one of the exams.

Schmidt

Exercise 1

Make table showing affordability & revealed preferences. Get w by Wales Law.

t	t'	$p^t x^t$		w^t	reveled preference
0	1	96	$>$	84	-
	2	80	$<$	84	$x^0 > x^2$
1	0	33	$<$	36	$x^1 > x^0$
	2	39	$>$	36	-
2	0	52	$>$	50	-
	1	48	$<$	50	$x^2 > x^1$

(a) Violation of WARP occurs if $p'x \leqslant w'$ and $px' \leqslant w$

As we see in the table, this does not occur.

Therefore, WARP is satisfied.

(b) Looking at the last column, we find

$$x^0 > x^2 \text{ and } x^1 > x^0$$

Transitivity implies $x^1 > x^2$. But this is violated by the last rows of the table.

Exercise 2

(a) Apply Roy's identity:

$$x_1(p, w) = -\frac{\frac{\partial v(p,w)}{\partial p_1}}{\frac{\partial (p,w)}{\partial w}} = -\frac{-\frac{w}{p_1^2}}{\frac{1}{p_1} + \frac{1}{p_2}} = \frac{w}{p_1} \frac{1}{1 + p_1/p_2}$$

(b) (1) Invert $v(p, w)$. In equilibrium:

$$v(p, w) = u \quad ; \quad w = e(p, w)$$

$$e(p, u) = u \left[\frac{1}{p_1} + \frac{1}{p_2} \right]^{-1}$$

(2) Apply Shepherd's Lemma:

$$\begin{aligned} h_1(p, u) = \frac{\partial e(p, u)}{\partial p_1} &= u(-1) \left[\frac{1}{p_1} + \frac{1}{p_2} \right]^{-2} (-1) \left(\frac{1}{p_1} \right)^2 \\ &= \frac{u}{(1 + p_1/p_2)^2} \end{aligned}$$

(c)

$$x_1 (\lambda p_1 \lambda w) = \frac{\lambda w}{\lambda p_1} \frac{1}{1 + \frac{\lambda p_1}{\lambda p_2}} = \frac{w}{p_1} \frac{1}{1 + p_1/p_2} = x_1 (p_1 w)$$

Yes, it is.

(d) Let $f(\cdot)$ be such a monotonic transformation and apply it to $v(p, w)$. Then use Roy's identity:

$$\begin{aligned} \tilde{x}_l(p, w) &= - \frac{\frac{\partial f(v(p,w)}{\partial p_l}}{\frac{\partial f(v(p,w)}{\partial w}} \overset{*}{=} - \frac{\frac{\partial f(v(p,w))}{\partial v(p,w)} \frac{\partial v(p,w)}{\partial p_i}}{\frac{\partial f(v(p,w))}{\partial v(p,w)} \frac{\partial v(p,w)}{\partial w}} \\ &= - \frac{\frac{\partial v(p_1 w)}{\partial p_1}}{\frac{\partial v(p,w)}{\partial w}} = x_l(p, w) \end{aligned}$$

The step at * uses the chain rule to expand the expression. We see that the Walrasian demand remains the same, irrespective of the transformation.

Exercise 3

(a) <u>if:</u>

$$
\begin{aligned}
u(x) &= \alpha - \beta \exp(-cx) \\
u'(x) &= \beta c \cdot \exp(-cx) \\
u''(x) &= -\beta c^2 \exp(-cx) \\
r(x) &= -\frac{u''(x)}{u'(x)} = -\frac{-\beta c^2 \exp(-cx)}{\beta c \exp(-cx)} = c
\end{aligned}
$$

<u>only if:</u>

$$
\begin{aligned}
& c = -\frac{u''(x)}{u'(x)} = -\frac{\partial \ln (u'(x))}{\partial x} \\
& \Leftrightarrow \int_{\underline{x}}^{x} \frac{\partial \ln (u'(t))}{\partial t} dt = \int_{\underline{x}}^{x} -c dt \\
& \Leftrightarrow \quad \ln \left(\frac{u'(x)}{u'(\underline{x})} \right) = -cx + c\underline{x} \\
& \Leftrightarrow \quad u'(x) = u'(\underline{x}) \exp(-cx + c\underline{x}) \\
& \Leftrightarrow \quad \int_{\underline{x}}^{x} u'(y) dy = \int_{\underline{x}}^{x} u'(\underline{x}) \exp(-cy + c\underline{x}) dy \\
& \Leftrightarrow u(x) - u(\underline{x}) = u'(\underline{x}) \exp(c\underline{x}) \frac{1}{-c} [\exp(-cx) - \exp(-c\underline{x})] \\
& \Leftrightarrow u(x) = u'(\underline{x}) \exp(c\underline{x}) \frac{1}{-c} [\exp(-cx) - \exp(-c\underline{x})] + u(\underline{x}) \\
& = \alpha - \beta \exp(-cx)
\end{aligned}
$$

(b) Investor maximizes expected utility:

$$
\max_a \int u(w - a + az) d \mp (z)
$$

Assume interior solution: obtain FOC:

$$
\int u'(w - a + az)(z - 1) dF(z) = 0
$$

Plug in $u(x)$. Use $u(x) = -\exp(-cx)$ as all positive affine transformations of $\alpha - \beta \exp(-cx)$ or allowed:

$$
\begin{aligned}
&\int c \cdot \exp(-c(w - a + az))(z-1)dF(z) = 0 \\
\Leftrightarrow &\underbrace{c \cdot \exp(-cw + ca)}_{\neq 0} \int \exp(-caz)(z-1)dF(z) = 0 \\
\Leftrightarrow &\int \exp(-caz)(z-1)dF(z) = 0 \qquad \text{(I)}
\end{aligned}
$$

Equation (I) defines $\bar{a}$ implicitly and it is completely independent of w. Thus, as w rises, $\bar{a}$ stays constant.

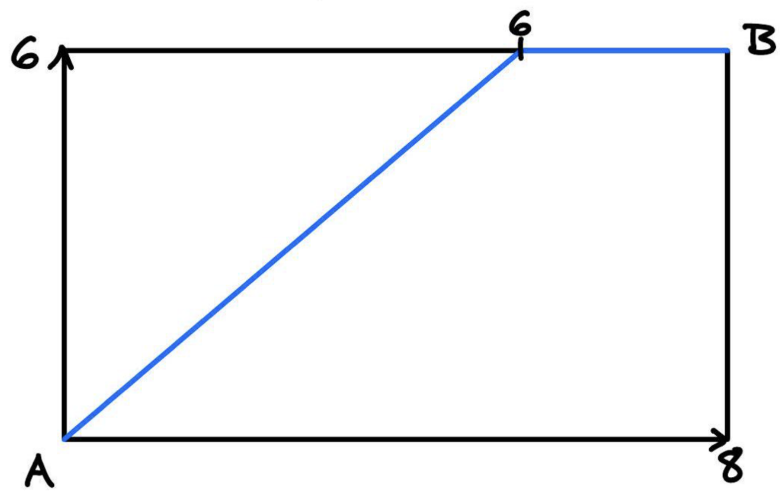
Gottardi

Exercise 1

(a)

$$MRS^A = 2\frac{x_2^A}{x_1^A} \stackrel{!}{=} MRS^B = 2 \Leftrightarrow x_2^A = x_1^A$$

PE allocations lie on $x_2^A = x_1^A$ until $x_1^A = 6$. From there $x_2^A = 6$.



(b)

$$U^A(w) = 2\ln(6) + \ln(2) = 4.277$$
$$U^B(w) = 2 \cdot 2 + 4 = 8$$

Try new allocation (going North-East):

$$u^A(5,3) = 2\ln(5) + \ln(3) = 4.317$$
$$u^B(3,3) = 2 \cdot 3 + 3 = 9$$

(c) Let $p = p_1/p_2$

<u>consumer A:</u>

$$\max_{x_1^A x_2^A} 2\ln\left(x_1^A\right) + \ln\left(x_2^A\right)$$
$$\text{s.t. } px_1^A + x_2^A = p6 + 2$$

FOCs

$$\left[x_1^A\right]: \quad 2\frac{1}{x_1^\lambda} - \lambda_p = 0$$
$$\left[x_2^A\right]: \quad \frac{1}{x_2^\lambda} - \lambda = 0$$
$$\longrightarrow x_2^A = p\frac{1}{2}x_1^A \tag{I}$$

<u>consumer B:</u>

$$\max_{x_1^P x_2^B} 2x_1^B + x_2^B$$
$$\text{st. } px_1^B + x_2^B = p_2 + 4$$

$$x_1^B = \begin{cases} \infty & \text{if} \quad p < 2 \\ \mathbb{R}^+ & \text{if} \quad p = 2 \\ 0 & \text{if} \quad p > 2 \end{cases} \quad x_2^B = \begin{cases} \infty & \text{if} \quad p > 2 \\ \mathbb{R}^+ & \text{if} \quad p = 2 \\ 0 & \text{if} \quad p < 2 \end{cases}$$

markets: For markets to cleo (no excess demand) must have $p = 2$.

Plug this into (I):

$$x_2^A = x_1^A \tag{II}$$

And this into BC for A :

$$x_1^A = \frac{14}{3} = x_2^A$$

<u>Market Clearing:</u>

$$\begin{aligned} x_1^B &= w_1^A + w_1^B - x_1^A = 8 - \frac{14}{3} = \frac{10}{3} \\ x_2^B &= w_2^A + w_2^B - x_2^A = 6 - \frac{14}{3} = \frac{4}{3} \end{aligned}$$

<u>Competitive Equilibrium:</u>

$$\begin{aligned} \left(x_1^A, x_2^A\right) &= \left(\frac{14}{3}, \frac{14}{3}\right) \\ \left(x_1^B, x_2^D\right) &= \left(\frac{10}{3}, \frac{4}{3}\right) \\ p &= 2 \end{aligned}$$

By (II) the CE is also PE.

(d) The price remains the same. Only the MRS of B is important for the price. If $p \neq 2$, then markets would not clear. Changing endowments does not affect the MRS of B. B's MRS is only so important because she has linear preferences, making the goods perfect substitutes for her.

(e) Equation (I) is now also valid for B. Sum over agents to see:

$$\begin{aligned} x_2^A + x_2^B &= p\frac{1}{2}\left(x_1^A + x_1^B\right) \\ p &= 2\frac{x_2^A + x_2^B}{x_1^A + x_1^B} \end{aligned}$$

By market clearing:

$$p = 2\frac{w_2^A + w_2^B}{w_1^A + w_1^B}$$

Therefore, the increase in w_1^B decreases P.

Exercise 2

$$w = (2, 4)$$

(a)

$$r_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad r_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Budget constraints:

$$a_1\theta_1 + a_2\theta_2 = 0 \tag{III}$$
$$x_1 = w_1 + 2\theta_1 + \theta_2 \tag{IV}$$
$$x_2 = w_2 + 2\theta_1 + 3\theta_2 \tag{V}$$

Solve consumer problem:

$$\max_{x_1, x_2} \frac{1}{2}\left(\sqrt{x_1} + \sqrt{x_2}\right)$$
$$\text{sit. (III) , (IV) , (V)}$$

Plug (IV) and (V) into objective function:

$$\max_{\theta_1\theta_2} \frac{1}{2}\left(\sqrt{w_1 + 2\theta_1 + \theta_2} + \sqrt{w_2 + 2\theta_1 + 3\theta_2}\right)$$
$$\text{sit. } q_1\theta_1 + q_2\theta_2 = 0$$

FOCs:

$$\frac{1}{4}\left[\frac{2}{\sqrt{w_1 + 2\theta_1 + \theta_2}} + \frac{2}{\sqrt{w_2 + 2\theta_1 + 3\theta_2}}\right] - \lambda q_1 = 0$$
$$\frac{1}{4}\left[\frac{1}{\sqrt{w_1 + 2\theta_1 + \theta_2}} + \frac{3}{\sqrt{w_2 + 2\theta_1 + 3\theta_2}}\right] - \lambda q_2 = 0$$

By market clearing and there only being one consumer we can set $\theta_1 = \theta_2 = 0$:

$$
\begin{aligned}
&\frac{1}{4}\left[\frac{2}{\sqrt{w_1}}+\frac{2}{\sqrt{w_2}}\right]-\lambda q_1=\frac{1}{4}[\sqrt{2}+1]-\lambda q_1=0\\
&\frac{1}{4}\left[\frac{1}{\sqrt{w_1}}+\frac{3}{\sqrt{w_2}}\right]-\lambda q_2=\frac{1}{4}\left[\frac{\sqrt{2}}{2}+\frac{3}{2}\right]-\lambda q_2=0\\
&\longrightarrow \frac{q_1}{q_2}=\frac{1+\sqrt{2}}{3+\sqrt{2}}2
\end{aligned}
$$

<u>Competitive Equilibrium:</u>

The CE is a non-trade equilibrium. As preferences are convex, the CE is unique.

$$
\begin{aligned}
(x_1,x_2)&=(2,4)\\
(\theta_1,\theta_2)&=(0,0)\\
\frac{q_1}{q_2}&=2\frac{1+\sqrt{2}}{3+\sqrt{2}}
\end{aligned}
$$

(b) Note: in reality, we only know q_1/q_2. Thus we can only compare the expected rates of return but not calculate their absolute values:

$$
\begin{aligned}
\frac{a_1}{q_2}&=2\frac{1+\sqrt{2}}{3+\sqrt{2}}>1=\frac{\mathbb{E}(r_1)}{\mathbb{E}(r_2)}\\
\Rightarrow\frac{\mathbb{E}(r_1)}{q_1}&<\frac{\mathbb{E}(r_2)}{q_2}
\end{aligned}
$$

Asset two has the higher rate of expected return. We already know that this is only due to a higher relative price of asset 1. Asset 1 has to be more expensive, otherwise the consumer would buy it to insure herself against poorer state 1 as she is strictly risk-averse. She is alone in the market & thus the excess demand for asset 1 increases its relative price.

(c) No. The market was already complete. The new asset is a linear combination of the others and it introduces no new choice option for the consumer.

8 Microeconomics Midterm 2017 / 18

Schmidt

Exercise 1

(a) Since $5 \in [0, 7.5]$, this violates WARP.

See below in ex (b) why this is true.

(b) To violate WARP must find:

$$\left| \begin{array}{l} p'x \leq w' \\ px' \leq w \end{array} \right|$$

Note that we find w & w' by Walras Law:

$$\left| \begin{array}{l} 2 \cdot 10 + 4 \cdot y \leq 50 \\ 6 \cdot 5 + 3 \cdot 10 \leq 60 + 3 \cdot y \end{array} \right|$$

$$\Leftrightarrow \left| \begin{array}{r} 20 + 4y \leqslant 50 \\ 60 \leqslant 60 + 3y \; | \end{array} \right.$$

$$\Longleftrightarrow \left| \begin{array}{l} y \leqslant 7.5 \\ 0 \leqslant y \end{array} \right|$$

Thus WARP is violated if $y \in [0, 7.5]$.

Exercise 2

(a) Budget constraint:

$$px_1 + x_2 = w \quad \mid \text{ use } x_1 = \frac{1}{p}$$

$$\Leftrightarrow \quad x_2 = w - 1$$

(b) This has to be quasi-lineor:

$$\begin{aligned} v(p,w) &= f\left(x_1(p,w)\right) + x_2(p,w) \\ &= f\left(\frac{1}{p}\right) + w - 1 \end{aligned}$$

By Roy's identity:

$$\begin{array}{lccl} x_1 = & -\frac{\frac{\partial v(p_n)}{\partial p}}{\frac{\partial v}{\partial w}} = -\frac{f'\left(\frac{1}{p}\right)(-1)\left(\frac{1}{p}\right)^2}{1} & \overset{!}{=} & \frac{1}{p} \\ \Leftrightarrow & f'\left(\frac{1}{p}\right) = p & & \text{use } p = \frac{1}{x_1} \\ \Leftrightarrow & f'\left(x_1\right) = \frac{1}{x_1} & & \\ \Leftrightarrow & \int_{\underline{x}_1}^{x_1} f'(t)dt = \int_{\underline{x}_1}^{x_1} \frac{1}{t}dt & & \end{array}$$

$$\Longleftrightarrow \quad f\left(x_1\right) - f\left(\underline{x}_1\right) = \ln\left(x_1\right) - \ln\left(\underline{x}_1\right)$$

Thus:

$$f\left(x_1\right) = \ln\left(x_1\right) + \alpha$$

For simplicity, let $\alpha = 0$, as utility is ordinal. Find:

$$v(p,w) = \ln\left(\frac{1}{p}\right) + w - 1$$

(c) Immediately from (b):

$$u\left(x_1, x_2\right) = \ln\left(x_1\right) + x_2$$

Exercise 3

Cash constraints turn this problem into a consumer problem.

(a) Treat $R(\cdot)$ as indirect utility & C as wealth.

Apply Roy's identity:

$$z_1\left(p_1 w_1, w_2\right) = -\frac{\frac{\partial R(\cdot)}{\partial w_1(\cdot)}}{\frac{\partial R}{\partial C}} = -\frac{p(-\alpha)1/w_1}{p \cdot 1/C} = \alpha \frac{C}{w_1}$$

(b) Invert $R(\cdot)$ to find $C(\cdot)$:

$$\begin{aligned}
&\frac{R}{p} = \gamma + \ln\left[\frac{C}{w_1^{\alpha} w_2^{1-\alpha}}\right] \\
&\Leftrightarrow \quad \exp\left[\frac{R}{P} - \gamma\right] = \frac{C}{w_1^{\alpha} w_2^{1-\alpha}} \\
&\Leftrightarrow C\left(p, w_1, w_2\right) = w_1^{\alpha} w_2^{1-\alpha} \exp\left[\frac{R}{p} - \gamma\right] \qquad \text{(I)}
\end{aligned}$$

(c) Apply Shepherd's Lemma to (I):

$$\tilde{z}_1\left(p_1 w_1, w_2\right) = \frac{\partial C(\cdot)}{\partial w_1} = \alpha\left(\frac{w_2}{w_1}\right)^{1-\alpha} \exp\left[\frac{R}{p} - \gamma\right]$$

Exercise 4

(a) (1) risk aversion: $u''(x) = 2c < 0 \Longleftrightarrow c < 0$

(2) marginal utility must be positive:

$$\begin{aligned}
u'(x) = b + 2cx \quad &> 0 \\
\Rightarrow \quad b \quad &> 2x|c|
\end{aligned}$$

There is no restriction on a. a only shifts the utility up or down.

(b) To satisfy (2) from (a), must have

$$x \in \left[0, -\frac{b}{2c}\right]$$

(c)

$$\begin{aligned}
EU(x) &= \int a + bx + cx^2 dF(x) \\
&= a + b\int xdF(x) + c\int x^2 dF(x) \\
&= a + b\mathbb{E}(x) + c\mathbb{E}\left(x^2\right) \quad | \text{ let } \mathbb{E}(x) = \mu \text{ and } Var(x) = \sigma^2 \\
&= a + b\mu + c\left(\sigma^2 + \mu^2\right)
\end{aligned}$$

(d)

$$\max_s \int a + b(w - s + sr) + c(w - s + sr)^2 dF(r)$$

Note:

$$\begin{aligned}
\mathbb{E}(w - s + sr) = w - s + s\mathbb{E}(r) = w - s + s\mu r \\
\operatorname{Var}(w - s + sr) = s^2 \operatorname{Var}(r) = s^2 G_r^2
\end{aligned}$$

Thus:

$$\max_s a + b\left(w - s + s\mu_r\right) + c\left[s^2 G_r^2 + \left(w - s + s\mu_r\right)^2\right]$$

FOC:

$$\begin{aligned}
& b\left(\mu_r - 1\right) + c2\left[sG_r^2 + \left(w - s + s\mu_r\right)\left(\mu_r - 1\right)\right] = 0 \\
& sG_r^2 + \omega\left(\mu_r - 1\right) + s\left(\mu_r - 1\right)^2 = \left(1 - \mu_r\right)\frac{b}{2c} \\
& s\left[\sigma_r^2 + \left(\mu_r - 1\right)^2\right] = \left(1 - \mu_r\right)\left(\frac{b}{2c} + \omega\right)
\end{aligned}$$

$$s^* = \frac{(1-\mu_r)\left(\frac{b}{2c}+\omega\right)}{\sigma_r^2 + (\mu_r - 1)^2}$$

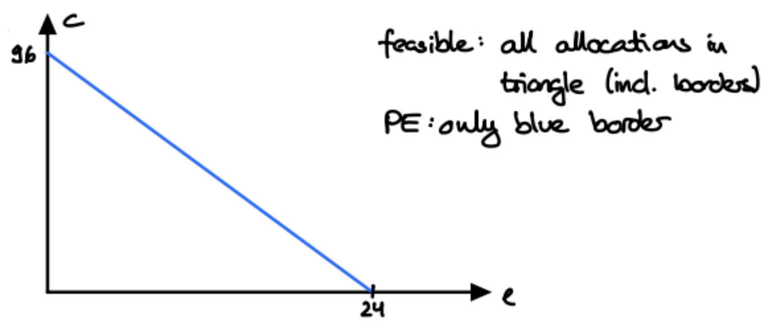
and therefore we find that

$$\frac{\partial s^*}{\partial w} = \frac{1-\mu_r}{\sigma_r^2 + (\mu_r - 1)^2} < 0$$

Gottardi

Exercise 1

(a) Figure says it all



(b) <u>Consumer:</u>

$$\max_{c,l} cl$$
$$\text{s.t. } pc + wl = w24$$
$$\iff \max_{c} c\left[24 - \frac{p}{wc}\right]$$

FOC:

$$
\begin{aligned}
& 24 - 2\frac{P}{wc} = 0 \\
\Leftrightarrow \quad & c = 12(p/w)^{-1}
\end{aligned}
$$

<u>Firm:</u>

$$\max_L p4L - wL = \max_L L\,(4p - w)$$

$$L = \begin{cases} \infty & \text{if} \quad p/w > 1/4 \\ \mathbb{R}^+ & \text{if} \quad p/w = 1/4 \\ 0 & \text{if} \quad p/w < 1/4 \end{cases}$$

<u>Markets:</u>

To clear labour market, must have $p/w = 1/4$ or there will be excess demand or supply.

$$\longrightarrow c = 12(p/w)^{-1} = 48$$

to clear goods market: $c = y = 48$

$$\longrightarrow l = 24 - \frac{p}{w}c = 12$$

by labour market: $L = 24 - l = 12$

<u>Competitive Equlibrium:</u>

$$\frac{p}{w} = \frac{1}{4}; y = 48; L = 12$$

This allocation is PE as it is on the border of the blue triangle described by

$$c = 24 - \frac{1}{4}l$$

(c) The price ratio is the same or labour market cannot clear.

<u>Consumer:</u>

$$\begin{aligned}
&\max_{c,l} cl - \bar{y} \\
&\text{s.t. } \bar{y} = 4(24 - l) \\
&pc + wl = 24w \\
&\Leftrightarrow \max_{l} \left(24(p/w)^{-1} - l(p/w)^{-1}\right) l - 4(24 - l)
\end{aligned}$$

FOC:

$$\begin{aligned}
&24(p/w)^{-1} - 2l(p/w)^{-1} + 4 = 0 \\
&12 + 1 - l = 0 \\
&l = 13
\end{aligned}$$

<u>Markets:</u>

$$\begin{aligned}
L &= 24 - l = 11 \\
c &= y = 44
\end{aligned}$$

<u>Competitive Equilibrium:</u>

$$\frac{p}{w} = \frac{1}{4}; L = 11; y = 44$$

This is not PE anymore.

Exercise 2

No. Only need LNS by FWT. But without convexity we may not have a CE at all.

Exercise 3

(a)

$$\begin{aligned} t = 0: \quad & q_1\theta_1 + q_2\theta_2 = 0 \\ t = 1\,;\ s = 1: \quad & x_1 = \theta_1 + 8 \\ t = 1\,;\ s = 2: \quad & x_2 = \theta_2 + 2 \end{aligned}$$

Combine:

$$q_1(x_1 - 8) + q_2(x_2 - 2) = 0$$

(b)

$$\begin{aligned} & \max_{x_1,x_2} \frac{1}{2}\left[10x_1 - \frac{1}{2}x_1^2 + 10x_2 - \frac{1}{2}x_2^2\right] \\ & \text{s.t. } q_1(x_1 - 8) + q_2(x_2 - 2) = 0 \end{aligned}$$

FOCs:

$$\begin{aligned} [x_1]: \quad & \frac{1}{2}(10 - x_1) - \lambda q_1 = 0 \\ [x_2]: \quad & \frac{1}{2}(10 - x_2) - \lambda q_2 = 0 \end{aligned}$$

Combine FOCs:

$$\frac{q_1}{q_2} = \frac{10 - x_1}{10 - x_2}$$

Asset market clearing implies $\theta_1 = \theta_2 = 0$ as there is only one consumer. As a result: $x_1 = w_1$ and $x_2 = w_2$.

$$\frac{q_1}{q_2} = \frac{10-8}{10-2} = \frac{2}{8} = \frac{1}{4}$$

We find $q_1 < q_2$, although $\mathbb{E}\,(r_1) = \mathbb{E}\,(r_2) = 1/2$. The reason is that the risk averse consumer wants to insure against the poor state (2) by buying asset 2. But as she is alone this creates excess demand for asset 2. This drives the price up until the consumer does not want to boy or sell.

(c) No. By removing risk aversion, the prices will reflect the state probabilities as the consumer only cares about expected payoff. Thus $q_1 = q_2$ and $\frac{q_1}{q_2} = 1$.

9 Microeconomics Midterm 2018 / 19

Schmidt

Exercise 1

(a) Violation of WARP:

$$\begin{vmatrix} py' \leqslant w \\ p'y \leqslant w' \end{vmatrix}$$

Plug in the prices & incomes (by Walras Law):

$$\begin{vmatrix} 30(12+x) & \leqslant 600 \\ 540 & \leqslant 360 + 24x \end{vmatrix}$$
$$\Leftrightarrow \begin{vmatrix} x \leq 8 \\ 7.5 \leqslant x \end{vmatrix}$$

Thus, WARP is violated if $x \in [7.5, 8]$.

(b) For this, bundle from year 2 must be affordable in year 1:

$$x \leq 8$$

But we exclude all x for which WARP is violated and find: $x \in [0, 7.5)$

(c) The quantity has increased, so the income must have decreased to find $\frac{\partial y_2}{\partial w} < 0$:

$$600 < 360 + 24x$$
$$\Leftrightarrow \quad 10 < x$$

Exercise 2

(a) Use hint because if $g(h(\cdot))$ represents preferences, then any strictly monotone transformation does it as well. Thus $h(\cdot)$ represents the preferences & is homogeneous. Call $h(\cdot)$ now $u(\cdot)$:

EMP:

$$\begin{aligned} &\min_x px \\ &\text{s.t. } u(x) = 1 \end{aligned}$$

FOC:

$$p_l - \lambda \frac{\partial u(x)}{\partial x_l} = 0 \quad \forall l$$

by Euler

$$e(p, u = 1) = \sum_l p_l x_l = \lambda \sum_l x_l \frac{\partial u(x)}{\partial x_l} = \lambda$$

now let $u(x) = u$:

$$\begin{aligned} &\min_x px \\ &\text{s.t. } u(x) = u \end{aligned}$$

FOC:

$$p_l - \lambda \frac{\partial u(x)}{\partial x_l} = 0 \quad \forall l$$
$$e(p,u) = \sum_l p_l x_l = \lambda \sum_l x_l \frac{\partial u(x)}{\partial x_l} = \lambda u = u \cdot e(p)$$

(b) UMP:

$$\max_x u(x)$$
$$\text{st. } px = 1$$

FOC:

$$\frac{\partial u(x)}{\partial x_l} - \lambda p_l = 0 \quad \forall l$$
$$\Leftrightarrow \frac{\partial u(x)}{\partial x_l} = \lambda p_l \quad \mid \cdot x_l$$
$$\frac{\partial u(x)}{\partial x_l} x_l = \lambda p_l x_l$$

Sum over x_l and use Euler:

$$\sum_l \frac{\partial u(x)}{\partial x_l} x_l = \lambda \sum_l p_l x_l \tag{I}$$
$$u(x^*) = v(p) = \lambda px = \lambda$$

Let $px = 1$: Same FOC and up to (I) nothing changes:

$$\sum_l \frac{\partial u(x)}{\partial x_l} x_l = \lambda \sum_l p_l x_l$$
$$u(x^*) = v(p,w) = \lambda px = \lambda w = v(p)w \tag{II}$$

(c) Follows from applying Roy's identity to (II):

$$x_l(p,w) = -\frac{\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial v p_1 w)}{\partial w}} = -\frac{\frac{\partial v(p)}{\partial p_l} w}{v(p)} = x_l(p)w$$

Exercise 3

(a) Leontief implies: $x_1^* = x_2^*$ and $u = x_1^* = x_2^*$ Thus, they must be able to afford the old bundle as Leontief does not allow for substitutions. They will also choose to consume it.

Before moving: $p_1 = p_2 = 1$

$$\rightarrow \quad x_1^* = x_2^* = \frac{w}{2} = 500 = u_0$$

After moving: Set $u_1 = u_0$. Thus

$$x_1^* = x_2^* = 500$$

to afford this:

$$e(p,u) = 500(1+4) = 2500$$

As initial wage is 1000, we have $R = 1500$.

This is the negative of CV.

(b) This is Cobb Douglas utility. Thus

$$x_1^* = \frac{w}{2p_1} \quad ; \quad x_2^* = \frac{w}{2p_2}$$

Before moving:

$$x_1^* = x_2^* = 500$$
$$v(p,w) = 500$$

After moving:

$$v(p,w) = (w+R)\left(\frac{1}{2p_1}\right)^{1/2}\left(\frac{1}{2p_2}\right)^{1/2} = 500$$

$$\Leftrightarrow \quad w + R = 2000 \Leftrightarrow R = 1000$$

As CD utility allows for substitution, they choose to buy less of x_1 as it has become much more expensive.

Exercise 4

(a) This agent exhibits decreasing absolute risk aversion.

$$r^A = -\frac{u''(x)}{u'(x)} = -\frac{-\rho\,(1_{-\rho})_x - \rho - 1}{(1-\rho)_x^{-\rho}} = \rho x^{-1}$$

(b) Agent maximizes expected utility:

$$\max_a \int u(W - aW + aW\pi)dF(\pi)$$

Assume interior solution:

$$\text{FCC:} \qquad \int u'(w - aw + aW\pi)(\pi w - w)dF(\pi) = 0$$

Plug in functional form:

$$\int (1-\rho)(1-a+a\pi)^{-\rho}(\pi-1)w^{1-\rho}dF(\pi) = 0$$

$$\Leftrightarrow \underbrace{(1-\rho)w^{1-\rho}}_{\neq 0}\int (1-a+a\pi)^{-\rho}(\pi-1)dF(\pi) = 0$$

$$\Rightarrow \qquad \int (1-a+a\pi)^{-\rho}(\pi-1)dF(\pi) = 0$$

This expression implicitly defines a^* and is independent of W.

Gottardi

Exercise 1

(i) Consumer A:

$$\max_{x_1^A, x_2^A} \ln\left(x_1^A\right) + 2\ln\left(x_2^A\right)$$
$$\text{s.t. } px_1^A + x_2^A = 16p$$

FOC:

$$\left[x_1^A\right] : \frac{1}{x_1^A} - \lambda p = 0$$
$$\left[x_2^A\right] : 2\frac{1}{x_2^A} - \lambda = 0$$
$$\longrightarrow x_2^A = 2x_1^A p \longrightarrow x_1^A = \frac{16}{3}$$

Consumer B:

$$\max_{x_1^B x_2^B} \ln\left(x_1^B\right) + \ln\left(x_2^B\right)$$
$$\text{s.t. } px_1^B + x_2^B = 12$$

FOC:

$$\left[x_1^B\right] : \frac{1}{x_1^B} - \lambda p = 0$$
$$\left[x_2^B\right] : \frac{1}{x_2^B} - \lambda = 0$$
$$\longrightarrow x_2^B = x_1^B p \longrightarrow x_2^B = 6$$

Markets:

$$x_1^A + x_1^B = 16 \quad ; \quad x_2^A + x_2^B = 12$$

$$\Leftrightarrow x_1^B = 16\frac{2}{3} \quad \Leftrightarrow \quad x_2^A = 6$$

Combine with either FOC to find:

$$\begin{aligned} x_2^B &= p x_1^B \\ 6 &= p16\frac{2}{3} \\ p &= 9/16 \end{aligned}$$

<u>Competitive Equilibrium:</u>

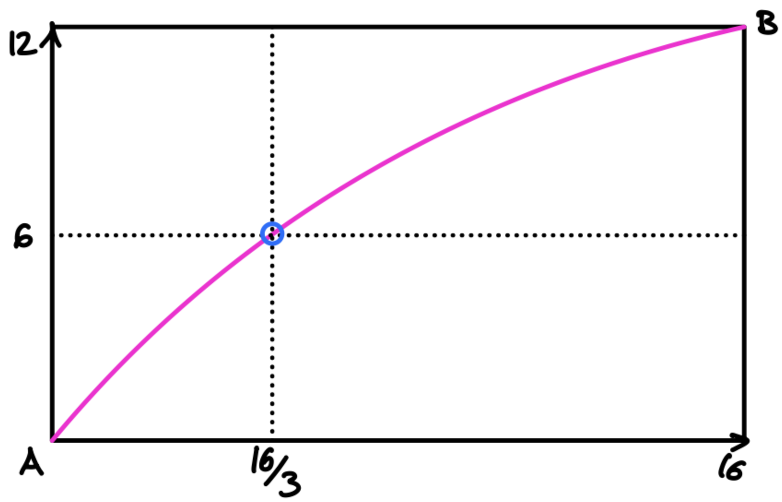
$$\begin{aligned} \left(x_1^A, x_2^A\right) &= (16 \cdot \frac{1}{3}, 6) \\ \left(x_1^B, x_2^B\right) &= (16 \cdot \frac{2}{3}, 6) \\ p &= 9/16 \end{aligned}$$

(ii)

$$MRS^A = \frac{x_2^A}{2x_1^A} \overset{!}{=} MRS^B = \frac{x_2^B}{x_1^B}$$

Use market clearing: $\quad x_1^B = 16 - x_1^A$

$$\begin{aligned} \longrightarrow \quad & \frac{x_2^A}{2x_1^A} = \frac{12 - x_2^A}{16 - x_1^A} \\ \Leftrightarrow \quad & \frac{16 - x_1^A}{2x_1^A} = \frac{12}{x_2^A} - 1 \\ \Leftrightarrow \quad & \frac{16 - x_1^A + 2x_1^A}{2x_1^A} = \frac{12}{x_2^A} \\ \Leftrightarrow \quad & x_2^A = \frac{24x_1^A}{16 + x_1^A} \qquad \text{(I)} \end{aligned}$$



(iii) Plugging into (I) :

$$8 = \frac{24 \cdot 8}{16 + 8}$$
$$8 = 8$$

Yes, it is PE.

Find transfers:

$$T^A = \begin{bmatrix} x_1^A \\ x_2^A \end{bmatrix} - \begin{bmatrix} w_1^A \\ w_2^A \end{bmatrix} = \begin{bmatrix} 8 \\ 8 \end{bmatrix} - \begin{bmatrix} 16 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 \\ 8 \end{bmatrix}$$
$$T^B = \begin{bmatrix} x_1^B \\ x_2^B \end{bmatrix} - \begin{bmatrix} w_1^B \\ w_2^B \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix} - \begin{bmatrix} 0 \\ 12 \end{bmatrix} = \begin{bmatrix} 8 \\ -8 \end{bmatrix}$$

Prices given by MRS:

$$p = MRS^A = MRS^B = 1/2$$

Exercise 2

(i) at $t = 0$:

$$q_1\theta_1 + q_2\theta_2 = 0$$

at $t = 1$ and $s = 1$:

$$x_1 = 2\theta_1 + \theta_2 + 4$$

at $t = 1$ and $s = 2$:

$$x_2 = \theta_1 + 2\theta_2 + 8$$

(ii) consumer solves:

$$\max_{\theta_1,\theta_2} \frac{1}{2}\left[\ln\left(2\theta_1 + \theta_2 + 4\right) + \ln\left(\theta_1 + 2\theta_2 + 8\right)\right]$$
$$\text{s.t. } q_1\theta_1 + q_2\theta_2 = 0$$

FOCs:

$$[\theta_1] : \left(2\theta_1 + \theta_2 + 4\right)^{-1} + \frac{1}{2}\left(\theta_1 + 2\theta_2 + 8\right)^{-1} - \lambda q_1 = 0$$
$$[\theta_2] : \frac{1}{2}\left(2\theta_1 + \theta_2 + 4\right)^{-1} + \left(\theta_1 + 2\theta_2 + 8\right)^{-1} - \lambda q_2 = 0$$

Suppose $q_1 = q_2 = 1$:

$$\frac{1}{2}\left(2\theta_1 + \theta_2 + 4\right)^{-1} = \frac{1}{2}\left(\theta_1 + 2\theta_2 + 8\right)^{-1}$$
$$2\theta_1 + \theta_2 + 4 = \theta_1 + 2\theta_2 + 8$$
$$\theta_1 = \theta_2 + 4 \tag{II}$$

Market clearing: $\theta_1 = -\theta_2 = 0$ as there is only one consumer. This violates (II). Thus $q_1 = q_2 = 1$ is not possible!

This result is the consequence of risk-aversion. The consumer is poorer in state 1, so she wants to buy insurance against it via asset 1. Unfortunately, she cannot because there is nobody else in the economy to trade with. To offset this excess demand for asset 1 we must have $q_1 > q_2$ which makes it less attractive.

(iii) There is no risk aversion and therefore the assets are not interesting as an insurance as they have the same expected return. As a consequence the prices reflect the state probabilities. As $\pi_1 = \pi_2 = 1/2$, will find $q_1 = q_2$ and $q_1 = q_2 = 1$ is a CE.

10 Microeconomics Midterm 2019 / 20

Schmidt

Exercise 1

(i) First, find incomes:

$$w^0 = 42 \quad w^1 = 36 \quad w^2 = 50$$

Look for violations of WARP:

t	t'	$p^t x^{t^1}$	$\sum$	w^t	revealed preferences
0	1	$p^0x^1 = 48$	$>$	42	-
	2	$p^0x^2 = 40$	$<$	42	$x^0 > x^2$
1	0	$p^1x^0 = 33$	$<$	36	$x^1 > x^0$
	2	$p^1x^2 = 39$	$>$	36	-
2	0	$p^2x^0 = 52$	$>$	50	-
	1	$p^2x^1 = 48$	$<$	50	$x^2 > x^1$

From the table we see that we never have $p^t x^{t'} \leq w^t$ and $p^{t'} x^t \leq w^{t'}$. Therefore, WARP is satisfied.

(ii) From the last row we have $x^0 > x^2$ and $x^2 > x^1$

Transitivity implies $x^0 > x'$ but we found the opposite: $x' > x^0$. Therefore, transitivity is violated.

Exercise 2

(a) Consumer 1: at optimum $e_1(\cdot) = w_1$ & $u_1 = v_1(\cdot)$

$$w_1 = v_1(p, w_1)\sqrt{p_1 p_2}$$
$$\Leftrightarrow v_1(p, w_1) = \frac{w_1}{\sqrt{p_1 p_2}}$$

Use Roy's identity:

$$x_1^1(p,w) = -\frac{\frac{\partial v_1(p_1 w)}{\partial p_1}}{\frac{\partial v_1(p_1 w)}{\partial w_1}} = -\frac{-\frac{1}{2}\frac{w_1}{\sqrt{p_2}p_1^{-3/2}}}{\frac{1}{\sqrt{p_1 p_2}}}$$
$$= \frac{w_1}{2p_1}$$

By symmetry: $x_2^1\left(p_1 w\right) = \frac{w_1}{2p_2}$

Consumer 2: Transform utility function.

$$u_2\left(x_1, x_2\right) = x_1^{\frac{3}{3+a}} x_2^{\frac{a}{3+a}}$$

This is standard Cobb-Dauglas:

$$x_1^2(p,w) = \frac{3}{3+a}\frac{w_2}{p_1}; x_2^1(p,w) = \frac{a}{3+a}\frac{w_2}{p_2}$$

(b) Good 1: $x_1^1 + x_1^2 = \frac{1}{p_1}\left[\frac{1}{2}w_1 + \frac{3}{3+a}w_2\right]$

$$\longrightarrow \frac{1}{2} = \frac{3}{3+a} \iff a = 3$$

Good 2: $x_2^1 + x_2^2 = \frac{1}{p_2}\left[\frac{1}{2}w_1 + \frac{a}{3+a}w_2\right]$

$$\longrightarrow \frac{1}{2} = \frac{a}{3+a} \iff a = 3$$

Thus $a = 3$ solves the problem for both goods.

Exercise 3

(a) Firm solves:

$$\min_x c(w,y) = \min_x wx$$
$$\text{s.t. } f(x) = y$$

FOC:

$$w_l = \lambda \frac{\partial f(x)}{\partial x_l} \quad \forall l$$

Use Euler:

$$c(w, y) = \lambda \sum_l \frac{\partial f(x)}{\partial x_l} x_l = \lambda f(x) = \lambda y$$

If $y = 1 : c(w, 1) = \lambda$

If $y \neq 1 : c(w, y) = \lambda y = c(w, 1)y = c(w)y$

(b) We have:

$$\begin{aligned} c(w, y) &= wx \\ \frac{\partial(w, y)}{\partial w_l} &= \frac{\partial wx}{\partial w_l} = x_l \end{aligned}$$

And from (a):

$$\frac{\partial c(w, y)}{\partial w_l} = \frac{\partial c(w, 1)}{\partial w_l} y$$

Together:

$$x_l = \frac{\partial c\,(w, 1)}{\partial w_l} y$$

(c) Profits are:

$$\pi = pf(x) - wx = pf(x) - \sum_l w_l x_l$$

Plug in the w_l from exercise

$$\begin{aligned}\pi &= pf(x) - \sum_l p\frac{\partial f(x)}{\partial x_l}x_l \\ &= p\left[f(x) - \sum_l \frac{\partial f(x)}{\partial x_l}x_l\right] = p[f(x) - f(x)]\end{aligned}$$

The last equality follows from CRS & Euler's formula. Clearly, $\pi = 0$.

Exercise 4

(i) DM maximize expected utility:

$$\begin{aligned}\max_{\alpha,\beta} EU(\cdot) &= \max_{\alpha,s} \int u(w - \alpha - \beta + \alpha z + \beta)dF(z) \\ &= \max_{\alpha} \int u(w - \alpha + \alpha z)dF(z)\end{aligned}$$

Get first order derivative:

$$\frac{\partial EU}{\partial \alpha} = \int u'(w - \alpha + \alpha z)(z-1)dF(z)$$

Suppose $\alpha = 0$:

$$\int u'(w)(z-1)dF(z) = u'(w)\left[\int zdF(z) - 1\right] > 0$$

As the expected marginal utility is positive at $\alpha = 0$, the DM will invert some $\alpha > 0$.

(ii) As we saw in (i), $\alpha = 0$ is not optimal (for both agents). They increase α, which lowers the marginal expected utility, until $\frac{\partial EU}{\partial \alpha} = 0$. Because $v(\cdot)$ is a concave transformation of $u(\cdot)$, we know that $v'(\cdot)$ decreases faster then $u'(\cdot)$. Therefore, $\int v'(\cdot)(z-1)dF(z) = 0$ is reached at a lower value of α than for $\int u'(\cdot)(z-1)dF(z)$. Thus:

$$\alpha_v^* < \alpha_u^*$$

Gottardi

Exercise 1

(i) Consumer A:

$$\max_{x_1^A, x_2} x_1^A x_2^A \text{ s.t. } px_1^A + x_2^A = p8$$

FOCs

$$\begin{aligned} \left[x_1^A\right] &: x_2^A - \lambda p = 0 \\ \left[x_2^A\right] &: x_1^A - \lambda = 0 \\ &\rightarrow x_2^A = px_1^A \end{aligned} \tag{I}$$

Combine (I) with BC:

$$x_1^A = 4 \quad ; \quad x_2^A = 4p$$

Consumer B:

$$\max_{x_1^B x_3^B} x_1^B + 2x_2^B \text{ s.t. } px_1^B + x_2^B = 6$$

By linearity:

$$x_1^B = \begin{cases} \infty & \text{if} \quad p < 1/2 \\ \mathbb{R}^+ & \text{if} \quad p = 1/2 \\ 0 & \text{if} \quad p > 1/2 \end{cases}$$

$$x_2^B = \begin{cases} \infty & \text{if} \quad p > 1/2 \\ \mathbb{R}^+ & \text{if} \quad p = 1/2 \\ 0 & \text{if} \quad p < 1/2 \end{cases}$$

<u>Markets:</u>

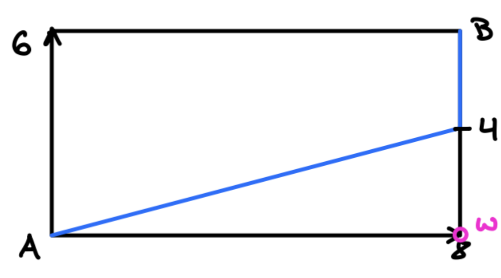
$p = 1/2$ otherwise we would have excess demand for one of the goods.

$$\longrightarrow x_2^A = 2$$
$$\longrightarrow x_1^B = 8 - 4 = 4$$
$$\longrightarrow x_2^B = 6 - 2 = 4$$

<u>Competitive Equilibrium:</u>

$$\left(x_1^A, x_2^A\right) = (4, 2)$$
$$\left(x_1^B, x_2^B\right) = (4, 4)$$
$$p = \frac{1}{2}$$

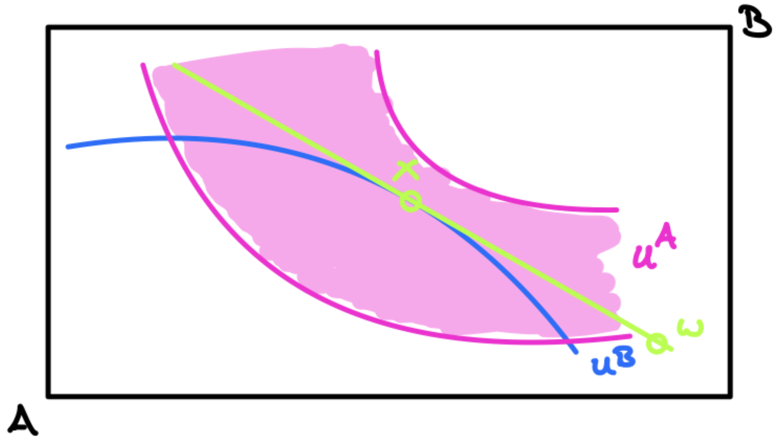
(ii) $MRS^A = x_2^A / x_1^A \stackrel{!}{=} MRS^B = 1/2 \longrightarrow x_2^A = \frac{1}{2} x_1^A$ in blue:



Exercise 2

1. LNS of preferences
2. complete markets
3. free disposal

Suppose (1) is violated. Then we could construct the following situation (A violates LNS):



Although at x both agents are optimizing given the prices, we could make B better off without hurting A if we moved to the bottom left. Thus the CE at x is not PE.

Exercise 3

$$(w_1, w_2) = (9, 16)$$

(a)

$$\begin{aligned} t = 0: & \quad q_1\theta_1 + q_2\theta_2 = 0 \\ t = 1 \text{ and } s = 1: & \quad x_1 = w_1 + \theta_1 + 3\theta_2 = 9 + \theta_1 + 3\theta_2 \\ t = 1 \text{ and } s = 2: & \quad x_2 = w_2 + 3\theta_1 + \theta_2 = 16 + 3\theta_1 + \theta_2 \end{aligned}$$

(b) Solve the maximization problem. I already substitute x_1 and x_2 from the BC s into the EU-function :

$$\max_{\theta_1\theta_2} \frac{1}{2}\left[\sqrt{9+\theta_1+3\theta_2}+\sqrt{16+3\theta_1+\theta_2}\right]$$
$$\text{st.} \quad q_1\theta_1+q_2\theta_2=0$$

FOC:

$$\frac{1}{2}\left[\frac{1/2}{\sqrt{9+\theta_1+3\theta_2}}+\frac{1/2\cdot 3}{\sqrt{16+3\theta_1+\theta_2}}\right]-\lambda q_1=0$$
$$\frac{1}{2}\left[\frac{1/2\cdot 3}{\sqrt{9+\theta_1+3\theta_2}}+\frac{1/2}{\sqrt{16+3\theta_1+\theta_2}}\right]-\lambda q_2=0$$

Since there is only one consumer. must have no trade equilibrium: $\theta_1=\theta_2=0$. Plug into FOCs:

$$\frac{1}{2}\left[\frac{1/2}{3}+\frac{1/2\cdot 3}{4}\right]-\lambda q_1=0 \Longleftrightarrow \lambda q_1=\frac{1}{4}\left[\frac{1}{3}+\frac{3}{4}\right]=\frac{13}{4\cdot 12}$$
$$\frac{1}{2}\left[\frac{1/2\cdot 3}{3}+\frac{1/2}{4}\right]-\lambda q_2=0 \Longleftrightarrow \lambda q_2=\frac{1}{4}\left[1+\frac{1}{4}\right]=\frac{5}{4\cdot 4}$$
$$\longrightarrow \frac{q_1}{q_2}=\frac{13}{12}\cdot\frac{4}{5}=\frac{13}{15}$$

(c) $\mathbb{E}\left(r_1\right)=\frac{1}{2}(1+3)=2=\mathbb{E}\left(r_2\right)=\frac{1}{2}(3+1)$

Thus:

$$\frac{q_1}{q_2}<1 \Longleftrightarrow \frac{1}{q_2}<\frac{1}{q_1} \Leftrightarrow \frac{\mathbb{E}\left(r_1\right)}{q_1}>\frac{\mathbb{E}\left(r_2\right)}{q_2}$$

The expected rate of return for asset 1 is larger than for asset 2.

Since the consumer is richer in state 2 and risk-averse, she would like to buy asset 2 as insurance. Because she is alone in the economy, this demand for asset 2 increases q_2 relative to q_1. This in turn leads to $\frac{1}{q_1}>\frac{1}{q_2}$ and $\frac{\mathbb{E}(r_1)}{q_1}>\frac{\mathbb{E}(r_2)}{q_2}$.

11 Microeconomics Midterm 2020 / 21

Schmidt

Exercise 1

(a) $x_1(\lambda p_1 \lambda w) = \lambda^{1+\alpha-\delta} \frac{p_1^\alpha w}{p_1^\delta + p_2^\delta + p_3^\delta} = \lambda^{1+\alpha-\delta} x_1(p, w)$

Must have $\alpha = \delta - 1$

$$x_2(\lambda p_1 \lambda w) = \lambda^{1+\alpha-\delta} \frac{p_2^\alpha w}{p_1^\delta + p_2^\delta + p_3^\delta} + \beta \frac{p_1}{p_3} \frac{\lambda}{\lambda}$$

No restriction on β.

$$x_3(\lambda p, \lambda w) = \lambda^{1+\alpha-\sigma} \frac{\gamma p_3^\alpha w}{p_1^\delta + p_2^\delta + p_3^\delta} = \lambda^{1+\alpha-\delta} x_3(p, w)$$

No restriction on γ.

In summary, we only need $\alpha = \delta - 1$

(b) $p_1 x_1(\cdot) + p_2 x_2(\cdot) + p_3 x_3(\cdot) = w$ to satisfy Walras' Law

$$\frac{w}{p_1^\delta + p_2^\delta + p_3^\delta} \left[p_1^{1+\alpha} + p_2^{1+\alpha} + \gamma p_3^{1+\alpha} \right] + \beta \frac{p_1 p_2}{p_3} = w$$

Must have $\beta = 0$:

$$p_1^\delta + p_2^\delta + p_3^\delta = p_1^{1+\alpha} + p_2^{1+\alpha} + \gamma p_3^{1+\alpha}$$

Must have $\gamma = 1$ & $\alpha = \delta - 1$.

In summary:

$$\alpha = \delta - 1 \quad \beta = 0 \quad \gamma = 1$$

Exercise 2

(1) Define $\tilde{\alpha}_1 = \frac{\alpha_1}{\alpha_1+\alpha_2}$ and $\tilde{\alpha}_2 = \frac{\alpha_2}{\alpha_1+\alpha_2}$. Then

$$u(x_1, x_2) = (x_1 - \gamma_1)^{\tilde{\alpha}_1} (x_2 - \gamma_2)^{\tilde{\alpha}_2}$$

and $\tilde{\alpha}_1 + \tilde{\alpha}_2 = 1$. This is allowed as it is a monotone transformation of utility.

(2) Define $\tilde{x}_1 = (x_1 - y_1)$ and $\tilde{x}_2 = (x_2 - y_2)$.

At the same time let $\tilde{w} = w - p_1 y_1 - p_2 y_2$.

The intuition is that we only allow the consumer to choose the excess consumption after obtaining at least γ_1 (or γ_2). For example, let x_1 be food and you need γ_1 food or you die. Thus you are only free to choose excess food after having γ_1. To make the budget work, I subtract the expenses for γ_1 (and γ_2) from the income.

(3) Now we get an immediate solution as the new problem is just standard Cobb-Douglas:

$$\begin{aligned}
&\max_{\tilde{x}_1, \tilde{x}_2} \tilde{x}_1^{\tilde{\alpha}_1} \tilde{x}_2^{\tilde{\alpha}_2} \\
&\text{s.t. } p_1 \tilde{x}_1 + p_2 \tilde{x}_2 = \tilde{w} \\
&\rightarrow \tilde{x}_1 = \tilde{\alpha}_1 \frac{\tilde{w}}{p_1} \quad ; \quad \tilde{x}_2 = \tilde{\alpha}_2 \frac{\tilde{w}}{p_2}
\end{aligned}$$

(4) Re-substitute:

$$\begin{aligned}
(x_1 - \gamma_1) &= \tilde{\alpha}_1 \frac{1}{p_1} (w - p_1 y_1 - p_2 \gamma_2) \\
\Leftrightarrow \qquad p_1 x_1 &= p_1 \gamma_1 + \tilde{\alpha}_1 (w - p_1 \gamma_1 - p_2 \gamma_2)
\end{aligned}$$

By symmetry:

$$p_2 x_2 = p_2 \gamma_2 + \tilde{\alpha}_2 (w - p_1 \gamma_1 - p_2 \gamma_2)$$

Exercise 3

(a) Cost is: $c(\cdot) = w_1 z_1(\cdot) + w_2 z_2(\cdot)$

Then:

$$\frac{\partial c(\cdot)}{\partial w_1} = z_1(\cdot) + w_1 \frac{\partial z_1(\cdot)}{\partial w_1} + w_2 \frac{\partial z_2(\cdot)}{\partial w_1} \tag{I}$$

The firm solves

$$\max_{z_1 z_2} pf(z_1, z_2) - w_1 z_1 - w_2 z_2$$

FOC:

$$p\frac{\partial f(\cdot)}{\partial z_1} - w_1 = 0 \Leftrightarrow w_1 = p\frac{\partial f(\cdot)}{\partial z_1} \tag{II}$$

$$p\frac{\partial f(\cdot)}{\partial z_2} - w_2 = 0 \Leftrightarrow w_2 = p\frac{\partial f(\cdot)}{\partial z_2} \tag{III}$$

Plug (II) and (III) into (I):

$$\begin{aligned}\frac{\partial c(\cdot)}{\partial w_1} &= z_1(\cdot) + p\frac{\partial f(\cdot)}{\partial z_1}\frac{\partial z_1(\cdot)}{\partial w_1} + p\frac{\partial f(\cdot)}{\partial z_2}\frac{\partial z_2(\cdot)}{\partial w_2} \\ &= z_1(\cdot) + p\left[\frac{\partial f(\cdot)}{\partial w_1} + \frac{\partial f(\cdot)}{\partial w_2}\right] = z_1(\cdot)\end{aligned}$$

(b) If production is below 2 units, then $c_1(y_1)$ is more cost efficient. Above 2 units, the firm can reduce cost by switching to $c_2(y_2)$:

$$c(y) = \begin{cases} y^2/2 & \text{if } y < 2 \\ y & \text{if } y \geqslant 2 \end{cases}$$

Exercise 4

(a) The agent maximizes expected utility:

$$\max_{A} \int u\left(w - A + Az\right) dF(z)$$

Getting the first coder derivative:

$$\frac{\partial EU}{\partial A} = \int u'(w - A + Az)(z-1)dF(z) \tag{IV}$$

Suppose $A = 0$:

$$\begin{aligned}\frac{\partial EU}{\partial A}(A=0) &= \int u'(w)(z-1)dF(2) \\ &= u'(w)\left[\int zdF(z) - 1\right] > 0\end{aligned}$$

As marginal expected utility is strictly positive, the agent would be marginally better off by investing $A > 0$. Thus, she would always do so.

(b) CARA implies $u(x) = \exp(-rx)$ since

$$-\frac{u''(x)}{u'(x)} = -\frac{r^2\exp(-rx)}{-r\exp(-rx)} = r$$

Go back to (IV) and set equal to zero for optimality condition:

$$\int u'(w - A + Az)(z-1)dF(z) = 0$$

Plug in $u'(x) = -r\exp(-rx)$:

$$
\begin{aligned}
&\int -r\exp(-r(w-A+Az))(z-1)dF(z) = 0\\
\Leftrightarrow\ &\underbrace{-r\exp(-r(w-A))}_{\neq 0}\int \exp\left(-rAz\right)(z-1)dF(z) = 0\\
\Leftrightarrow\ &\int \exp\left(-rAz\right)(z-1)dF(z) = 0 \qquad \text{(V)}
\end{aligned}
$$

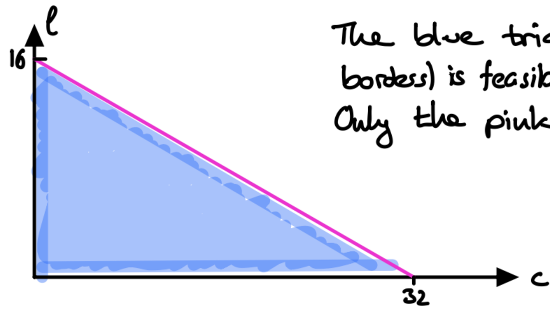
(V) implicitly defines the optimal A and does not depend on wealth.

Gottardi

Exercise 1

(a) Must have $c \leq y = 2L$ and $L = 16 - l$

Thus: $c \leqslant 32 - 2l$ describes feasible allocations. PE allocations are only at $c = 32 - 2l$, as otherwise, resources are wasted that could contribute towards utility:



The blue triangle (incl. borders) is feasible.
Only the pink border is PE.

(b) consumer:

$$
\max_{c,l} \ln(c) + \ln(l)
$$
$$
\text{st. } pc + wl = 16w
$$

FOCs:

$$\frac{1}{c} - \lambda p = 0$$
$$\frac{1}{c} - \lambda w = 0$$
$$\rightarrow c = \frac{w}{p} l$$

Firm:

$$\max PAL - wL$$

$$L = \begin{cases} \infty & \text{if} \quad A \geq w/p \\ \mathbb{R}^+ & \text{if} \quad A \geq w/p \\ 0 & \text{if} \quad A < w/p \end{cases}$$

Markets:

$$L = 16 - l \longrightarrow w/p = A = 2$$

$$\left| \begin{array}{l} c = y = AL = A(16 - l) = 32 - 2l \\ c = \frac{w}{p} l = Al = 2l \end{array} \right|$$
$$\longrightarrow l = 8; L = 8; c = y = 16$$

Competitive Equilibrium:

$$w/p = 2$$
$$y = 16$$
$$L = 8$$

Since $c = 32 - 2l$ holds, the CE is PE.

(c) (1) $\frac{w}{p}$ will increase, as $\frac{w}{p} = A'$. Else, we'd have $\frac{w}{p} < A'$ and the firm would demand infinite labour. This excess demand cannot exist in a CE.

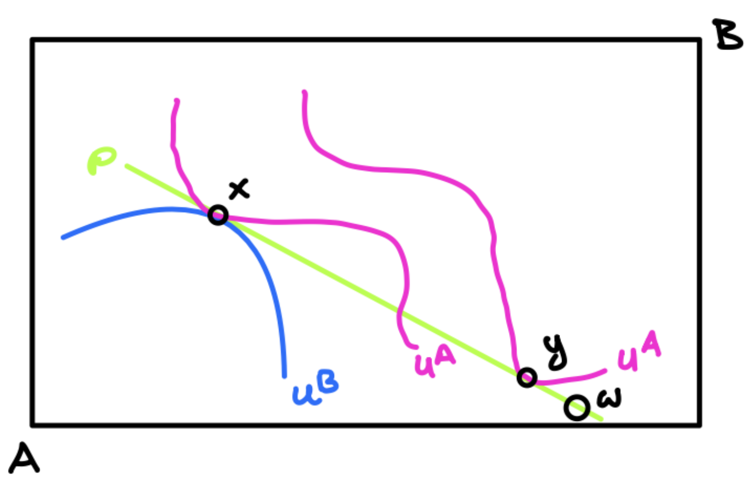
(2) y must increase. More productive firm increases its output.

(3) L remains the same. The firm produces more at a lower price and the consumer consumes more, working the same for a higher relative wage. She could work more and consume more but since $MRS = c/l$, this is not what happens.

The utility increases as $l = 8$ as before but c increases as y increases.

Exercise 2

Yes. If one of the consumers has non-convex preferences, we con find prices at PE allocations that are not CE:



Although x is PE, A could be better off at these prices. Therefore x is not a CE.

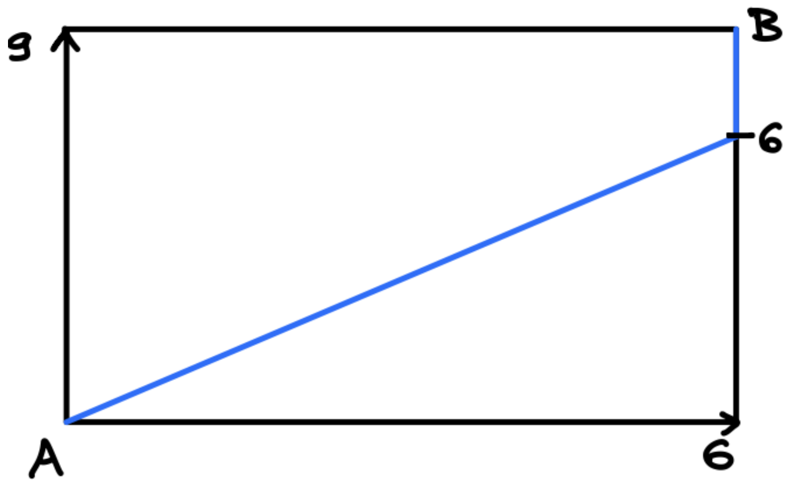
Exercise 3

$$w^A = (4,8) \quad ; \quad w^B = (2,1)$$

(a)

$$MRS^A = \frac{1/2x_2^A}{1/2x_1^A} \stackrel{!}{=} MRS^B = 1 \rightarrow x_2^A = x_1^A$$

All PE-allocation are in blue.



(b) <u>consumer A:</u>

$$\begin{aligned} &\max_{x_1^A, x_2^A} 1/2 \left(\ln\left(x_1^A\right) + \ln\left(x_2^A\right)\right) \\ \text{s.t. } & q_1\theta_1^A + q_2\theta_2^A = 0 \\ & x_1^A = w_1^A + \theta_1^A \\ & x_2^A = w_2^A + \theta_2^A \end{aligned}$$

$$\iff \quad \max_{x_1^A, x_2^A} 1/2\left(\ln\left(x_1^A\right) + \ln\left(x_2^A\right)\right)$$
$$\text{s.t. } q_1\left(x_1^A - w_1^A\right) + q_2\left(x_2^A - w_2^A\right) = 0$$

FOCs:

$$\left[x_1^A\right]: \frac{1}{2x_1^A} - \lambda q_1 = 0$$
$$\left[x_2^A\right]: \frac{1}{2x_2^A} - \lambda q_2 = 0$$
$$\rightarrow \frac{q_1}{q_2} = \frac{x_2^A}{x_1^A} \tag{I}$$

<u>consumer B:</u>

$$\max_{x_1^\beta x_2^\beta} 1/2\left(x_1^\beta + x_2^\beta\right)$$
$$\text{s.t. } q_1\left(x_1^B - w_1^D\right) + q_2\left(x_2^8 - w_1^B\right) = 0$$

FOCs:

$$\left[x_1^B\right]: 1/2 + \lambda q_1 = 0$$
$$\left[x_2^B\right]: 1/2 + \lambda q_2 = 0$$
$$\rightarrow \frac{q_1}{q_2} = 1 \tag{II}$$

Plug (II) into (I):

$$x_2^A = x_1^A \tag{III}$$

Use (II) and (III) in BC for A :

$$x_1^A = x_2^A = \frac{w_1^A + w_2^A}{2} = 6$$

markets:

$$\begin{aligned}
&x_1^A + x_1^B = w_1^A + w_1^B = 6 \\
&\longrightarrow x_1^B = 0 \\
&x_2^A + x_2^B = w_2^A + w_2^B = 9 \\
&\longrightarrow x_2^B = 3
\end{aligned}$$

Competitive Equilibrium:

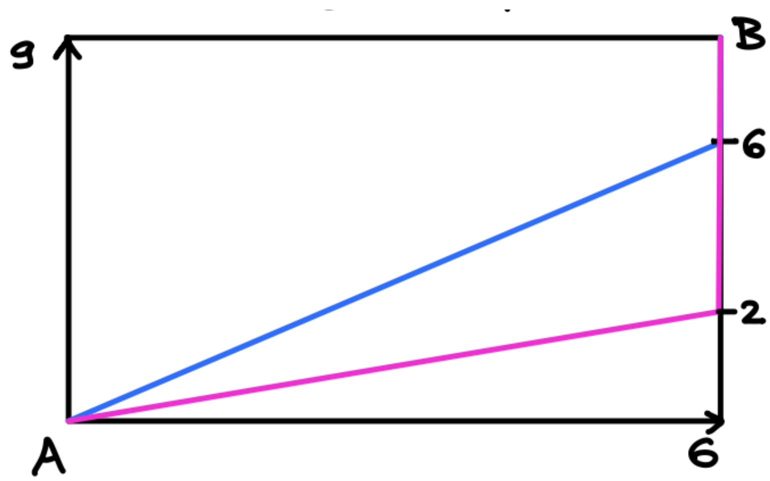
$$\begin{aligned}
\left(x_1^A, x_2^A\right) &= (6, 6) \\
\left(x_1^B, x_2^B\right) &= (0, 3) \\
\frac{q_1}{q_2} &= 1
\end{aligned}$$

(c) PE:

$$\begin{aligned}
MRS^A = x_2^A / x_1^A \stackrel{!}{=} MRS^B = 1/3 \\
\longrightarrow x_2^A = \frac{1}{3} x_1^A
\end{aligned}$$

The new set of PE-allocations is indicated in pink.

Now $\frac{q_1}{q_2} = \frac{1}{3}$. Reason being that the prices of the Arrow-securities reflect the state-probabilities of the risk-neutral agent as she will take on the entire risk in equilibrium.



12 Microeconomics Midterm 21 / 22

Schmidt

Exercise 1

(a) Clearly, WA is violated as $15 \in [0.22.5]$ by the result in (b).

(b) WA: if $x \neq x'$ and $p'x \leqslant w' \Rightarrow px' > w$

Thus check bundles in other price-wealth situations:

$$\left| \begin{array}{l} 4 \cdot 30 + 8 \cdot y = 120 + 8y \leqslant w_0 = 4 \cdot 15 + 8 \cdot 30 = 300 \\ 12 \cdot 15 + 6 \cdot 30 = 360 \leqslant w_1 = 12 \cdot 30 + 6y = 360 + 6y \end{array} \right|$$

$$\Leftrightarrow \left| \begin{array}{l} y \leqslant 22.5 \\ 0 \leqslant y \end{array} \right| \Longleftrightarrow y \in [0.22.5]$$

WA is violated if $y \in [0, 22.5]$.

Exercise 2

(a) Use Roy's identity:

$$x_l(p,w) = -\frac{\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial v(p,w)}{\partial w}}$$

Then let $f(\cdot)$ be a movotonic tranformation:

$$\tilde{x}_l(p,w) = -\frac{\frac{\partial f(v(p,w))}{\partial p_l}}{\frac{\partial f(p,w))}{\partial w}} = -\frac{\frac{\partial f(p,w)}{\partial v(p,w)}\frac{\partial v(p,w)}{\partial p_l}}{\frac{\partial f(p,w)}{\partial v(p,w)}\frac{\partial v(p,w)}{\partial w}} = x_l(p,w)$$

(b) (1) find $w(v(p,w))$:

$$w = \left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{1-\alpha}\right)^{1-\alpha} v(p,w)$$

At optimum: $w = e(p,u)$ and $v(p,w) = u$.

(2) Apply Shephard's Lemma to e(p,u):

$$\begin{aligned} h_1(p,u) = \frac{\partial e\left(p_1 u\right)}{\partial p_1} &= \alpha\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{p_1(1-\alpha)}\right)^{1-\alpha} u \\ &= \left(\frac{p_2}{p_1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha} u \end{aligned}$$

(c) case 1:

$$\begin{aligned} \alpha = \alpha\left(p_1/p_2\right) &\longrightarrow \alpha\left(\lambda p_1/\lambda p_2\right) = \alpha \\ h_1\left(\lambda p_1, u\right) &= \left(\frac{\lambda p_2}{\lambda p_1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha} u \\ &= \left(\frac{p_2}{p_1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha} u = h_1(p_1,u) \end{aligned}$$

case 2: $\alpha = \alpha\left(p_1\right)$

$$
\begin{aligned}
h_1(\lambda p_1, u) &= \left(\frac{\lambda p_2}{\lambda p_1}\frac{\alpha\left(\lambda p_1\right)}{1-\alpha\left(\lambda p_1\right)}\right)^{1-\alpha\left(\lambda_{p_1}\right)} u \\
&= \left(\frac{p_2}{p_1}\frac{\alpha\left(\lambda p_1\right)}{1-\alpha\left(\lambda p_1\right)}\right)^{1-\alpha\left(\lambda_{p_1}\right)} u \neq h_1\left(p_1, u\right)
\end{aligned}
$$

Exercise 3

The difference between consumer theory and production theory is mainly the fact that firms do not have budget constraints. This problem introduces a budget constraint. Therefore, we are going to treat the problem like a consumer problem. In that sense, the revenue is comparable to the utility function, and the cash constraint is like the wealth of a consumer. Consequently, we are solving the following revenue maximization problem (which is the analogue to a utility maximization problem):

$$
\begin{aligned}
&\max_{z_1, z_2} p f(z_1, z_2) \\
\text{s. t. } &\ w_1 z_1 + w_2 z_2 \leq C
\end{aligned}
$$

We will assume an interior solution (the budget constraint is binding). Then, the revenue function $R(p, w_1, w_2, C)$ that the exercise gives us is just the equivalent to the indirect utility.

(a) As $R(p, w_1, w_2, C)$ works like the indirect utility, we apply Roy's identity to find the factor demand, which is the analogue to the Walrasian demand:

$$
\begin{aligned}
z_1 &= -\frac{\frac{\partial R}{\partial w_1}}{\frac{\partial R}{\partial C}} \\
&= -\frac{p \cdot (-\alpha)\frac{1}{w_1}}{p \cdot \frac{1}{C}} \\
&= \alpha \frac{C}{w_1}
\end{aligned}
$$

(b) We treat $R(p,w,C)$ as the indirect utility depending on income and invert it to find the cost function $C(p,w,R)$, which is the analogue to the expenditure function in consumer theory:

$$\begin{aligned}
R &= p\left[\gamma + \ln C(p,w,R) - \alpha \ln w_1 - (1-\alpha)\ln w_2\right] \\
\frac{R}{p} - \gamma &= \ln\left(\frac{C(p,w,R)}{w_1^{\alpha} w_2^{1-\alpha}}\right) \\
\exp\left(\frac{R}{p} - \gamma\right) &= \frac{C(p,w,R)}{w_1^{\alpha} w_2^{1-\alpha}} \\
C(p,w,R) &= w_1^{\alpha} w_2^{1-\alpha} \exp\left(\frac{R}{p} - \gamma\right)
\end{aligned}$$

(c) Since the cost function from (b) happens to be the analogue to the expenditure function, we can apply Shephard's Lemma in order to find the factor demand for a given R at minimum cost, as this is the analogue to the Hicksian demand in consumer theory. In that spirit, let us call this function $h_1(p,w,R)$.

$$\begin{aligned}
h_1(p,w,R) &= \frac{\partial C(w,R)}{\partial w_1} \\
&= \alpha \exp\left[\frac{R}{p} - \gamma\right] \cdot \left(\frac{w_2}{w_1}\right)^{1-\alpha}
\end{aligned}$$

(d) In consumer theory, the Hicksian demand and the Walrasian demand meet at optimum. We can also show that here:

$$
\begin{aligned}
h_1(w, R) &= z_1^* \\
\alpha \exp\left[\frac{R}{p} - \gamma\right] \cdot \left(\frac{w_2}{w_1}\right)^{1-\alpha} &= \alpha \frac{C}{w_1} \\
\exp\left[\frac{R}{p} - \gamma\right] w_1^{\alpha} w_2^{1-\alpha} &= C \\
\frac{R}{p} - \gamma &= \ln\left(\frac{C}{w_1^{\alpha} w_2^{1-\alpha}}\right) \\
R &= p\left[\gamma + \ln C - \alpha \ln w_1 - (1-\alpha) \ln w_2\right]
\end{aligned}
$$

The last line is exactly the formula for the revenue that is observed by our econometrician friend in the optimum. Therefore, we have shown that the two demands are equal whenever the firm is acting optimally, i.e. maximizing its revenue or minimizing its cost. Put differently, the revenue maximization problem is the dual problem to the cost minimization problem and vice versa.

Exercise 4

(a) IF:

$u(x) = \beta x^{1-\rho} + \gamma$

$$
r^R = -x \frac{u''(x)}{u'(x)} = -x \frac{\beta(1-\rho)(-\rho)x^{-\rho-1}}{\beta(1-\rho)x^{-\rho}} = \rho
$$

ONLY IF:

$r^R = -x \frac{u''(x)}{u'(x)} = -x \frac{\partial \ln(u'(x))}{\partial x} = \rho$

$$
\begin{aligned}
\iff \quad & \frac{\partial \ln\left(u'(x)\right)}{\partial x} = -\rho \frac{1}{x} \\
\iff & \int_{\underline{x}}^{x} \frac{\partial \ln\left(u'(t)\right)}{\partial t} dt = -\rho \int_{\underline{x}}^{x} \frac{1}{t} dt \\
\iff \quad & \ln\left(u'(x)\right) - \ln\left(u'(\underline{x})\right) = -\rho(\ln(x) - \ln(\underline{x})) \\
\Leftrightarrow \quad & u'(x) = x^{-p} \frac{u'(\underline{x})}{\underline{x}^{-\rho}} = x^{-\rho}\alpha \\
& \int_{\underline{x}}^{x} u'(y) dy = \alpha \int_{\underline{x}}^{x} y^{-\rho} dy \\
\Leftrightarrow \quad & u(x) - u(\underline{x}) = \frac{\alpha}{1-\rho} \left(x^{1-p} - \underline{x}^{1-p}\right) \\
\Leftrightarrow \quad & u(x) = \beta x^{1-p} + \gamma
\end{aligned}
$$

Risk aversion:

$$
\begin{aligned}
& u''(x) < 0 \\
\Leftrightarrow & \beta(1-\rho)(-\rho)x^{-\rho-1} < 0 \\
\Rightarrow & \beta(1-\rho)\rho > 0
\end{aligned}
$$

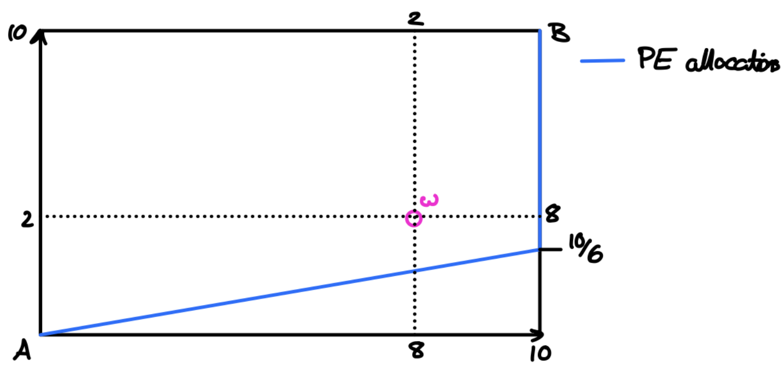
This only holds when ($\beta > 0$ and $\rho < 1$) or ($\beta < 0$ and $\rho > 1$).

Gottardi

Exercise 1

(i) Pareto Efficient:

$$MRS^A = 3\frac{x_2^A}{x_1^A} = MRS^B = \frac{1}{2}$$
$$\iff \quad x_2^A = \frac{1}{6}x_1^A$$



(ii) <u>consumer A:</u>

$$\max_{x_1^A, x_2^A} 3\ln\left(x_1^A\right) + \ln\left(x_2^A\right)$$
$$\text{s.t. } x_1^A + px_2^A = 8 + 2p$$

FOCs:

$$\left[x_1^A\right] : \frac{3}{x_1^A} - \lambda = 0$$
$$\left[x_2^A\right] : \frac{1}{x_2^A} - \lambda p = 0$$
$$\rightarrow x_1^A = 3px_2^A$$

<u>consumer B:</u> linear utility leads to:

$$x_1^B = \begin{cases} \infty & \text{if} \quad p \geq 2 \\ \mathbb{R}^+ & \text{if} \quad p = 2 \\ 0 & \text{if} \quad p < 2 \end{cases}$$

$$x_2^B = \begin{cases} \infty & \text{if } p \leq 2 \\ \mathbb{R}^+ & \text{if } p = 2 \\ 0 & \text{if } p > 2 \end{cases}$$

<u>market:</u>

$$x_1^A + x_1^B = 10 = x_2^A + x_2^B$$

In order for markets to clear with no excess demand, we must have $p = 2$ because of consumer B's preferences. Therefore

$$x_1^A = 6x_2^A$$

plug into BC^A : $\quad 8x_2^A = 8 + 4 \quad \iff x_2^A = 12/8 = 3/2$

$$\rightarrow x_1^A = 9 \rightarrow \left(x_1^B, x_2^B\right) = (1, 17/2)$$

<u>Competitive Equilibrium:</u>

$$\begin{aligned} \left(x_1^A, x_2^A\right) &= (9, 3/2) \\ \left(x_1^B, x_2^B\right) &= (1, 17/2) \\ \frac{p_2}{p_1} &= 2 \end{aligned}$$

Since $x_2^A = 1/6x_1^A$, PE is achieved.

(iii)

$$\begin{aligned}
u^A\left(w_1^A, w_2^A\right) &= 3\ln(8) + \ln(2) \cong 6.931 \\
u^A\left(x_1^A, x_2^A\right) &= 3\ln(9) + \ln(3/2) \cong 6.997 \\
u^B\left(w_1^B, w_2^B\right) &= 2 + 2 \cdot 8 = 18 \\
u^B\left(x_1^D, x_2^B\right) &= 1 + 2 \cdot 17/2 = 18
\end{aligned}$$

By FWT this is always true, when preferences do not violate LNS, there is free disposal and markets are complete.

Exercise 2

Autarky: A sells, B buys good 1.

Effect depends on price change & preferences.

Assume $\frac{P_1}{P_2}$ goes up (the other way round the argument can be reversed). This makes the seller better off as she gets more per unit sold and might even sell more. For B it depends on her preferences. If she can substitute and switch to selling good 1, she profits. If she has to buy good 1 at a higher price, she loses. It is also possible that her utility does not change despite the price change.

If prices remain the same, nothing changes.

Exercise 3

$$(w_1, w_2) = (1, 4)$$

(i) at $t = 0: \quad q_1\theta_1 + q_2\theta_2 = 0$

at $t = 1, s = 1: \quad x_1 = w_1 + \theta_1 4 + \theta_2$

at $t = 1, s = 2: \quad x_2 = w_2 + \theta_2$

(ii) Consumer problem:

$$\max_{\theta_1, \theta_2} 1/4\left(w_1 + 4\theta_1 + \theta_2\right)^{1/2} + 3/4\left(w_2 + \theta_2\right)^{1/2}$$

$$\text{s.t.} \quad q_1\theta_1 + q_2\theta_2 = 0$$

FOCs:

$$
\begin{aligned}
[\theta_1] &: \frac{4}{8\left(w_1+4\theta_1+\theta_2\right)^{1/2}} - \lambda q_1 = 0 \\
[\theta_1] &: \frac{1}{8\left(w_1+4\theta_1+\theta_2\right)^{1/2}} + \frac{3}{8\left(w_2+\theta_2\right)^{1/2}} - \lambda q_2 = 0
\end{aligned}
$$

<u>market clearing:</u>

$$
\theta_1 = -\theta_2 = 0 \tag{I}
$$

Plug (I) into FOCs:

$$
\begin{aligned}
&\frac{1}{2} = \lambda q_1; \quad \frac{1}{8} + \frac{3}{16} = \lambda q_2 \\
\longrightarrow &\frac{q_1}{q_2} = \frac{1}{2}\frac{16}{5} = \frac{8}{5}
\end{aligned}
$$

(iii) $\mathbb{E}\left(r_1\right) = \mathbb{E}\left(r_2\right) = 1$

$$
\longrightarrow \frac{\mathbb{E}\left(r_1\right)}{q_1} > \frac{\mathbb{E}\left(r_2\right)}{q_2} \Longleftrightarrow 1 > \frac{q_1}{q_2} = \frac{8}{5}
$$

We see that the inequality above is INCORRECT, we have run into a CONTRADICTION.

Usual intuition: $q_1/q_2 > 1$ because consumer wants to insure against poor state where she has less income. This leads to a lower expected rate of return for asset 1. Otherwise the consumer would buy asset 1 but she cannot because of market clearing.