Beginning Doctoral Program Gerzensee

Lectures held by Jordi Galí

Macroeconomics Finals Solutions

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Exercise 1

(a)

$$\mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(\cdot) + \frac{\lambda_t}{P_t} \left(B_{t-1} + W_t N_t + D_t - P_t C_t - Q_t B_t \right) \right\}$$

FOCs:

$$\begin{aligned}
&[C_t]\, \beta^t U_C(\cdot) - \lambda_t = 0 \Leftrightarrow \lambda_t = \beta^t U_C(\cdot) \\
&[N_t]\, \beta^t U_N(\cdot) + \lambda_t \frac{w_t}{R_t} = 0 \Rightarrow -\frac{U_N(\cdot)}{U_C(\cdot)} = \frac{w_t}{P_t} \\
&[B_t]\, \mathbb{E}_t \left(\lambda_{t+1} / P_{t+1} \right) - Q_t \left(\lambda_t / P_t \right) = 0
\end{aligned}$$

In logs (combine $[C_t]$ and $[N_t]$):

$$c_t + \varphi n_t = w_t - p_t \qquad \text{(intratemporal)}$$

Rewrite $[B_t]$:

$$Q_t = \beta \mathbb{E}_t \left[\; \tfrac{P_{t+1}}{P_t} \quad \tfrac{Z_{t+1}}{Z} \quad \tfrac{C_t}{C_{t+1}} \; \right]$$

In logs:

$$\begin{aligned}
-i_t &= -\rho + \mathbb{E}_t \left(\pi_{t+1} \right) + \left(\rho_z - 1 \right) z_t + c_t - \mathbb{E}_t \left(c_{t+1} \right) \\
c_t &= \mathbb{E}_t \left(c_{t+1} \right) - \left(i_t - \mathbb{E}_t \left(\pi_{t+1} \right) - \left(1 \rho_z \right) z_t - \rho \right) \qquad \text{(intertemporal)}
\end{aligned}$$

(b)

$$\left. \begin{aligned} &\text{Firms:} \quad p_t = w_t + \mu_t \\ &\text{HH:} \quad c_t + \varphi u_t = w_t - p_t \end{aligned} \right| \Rightarrow c_t + \varphi n_t = -\mu_t$$

$$\left.\begin{array}{ll}\text{markets clear:} & y_t = c_t \\ \text{technology:} & y_t = n_t\end{array}\right| \rightarrow c_t = n_t$$

All together:

$$\begin{aligned}(1+\varphi)y_t &= -\mu_t \\ y_t^n &= -\frac{\mu}{1+\varphi}\end{aligned}$$

Natural output level is constant in all periods. It is independent of z_t because it cancels in the intratemporal optimality condition. z_t influences C_t & N_t to the same degree!

(c) $\mu_t - \mu = -(1+\varphi)\tilde{y}_t$, where $\tilde{y}_t = y_t - y_t^n$

(d)

$$\begin{aligned}\tilde{y}_t &= \mathbb{E}_t\left(\tilde{y}_{t+1}\right) - \left(i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \left(1-\rho_z\right)z_t - \rho\right) \\ \pi_t &= \beta\mathbb{E}_t\left(\pi_{t+1}\right) + \lambda(1+\varphi)\tilde{y}_t\end{aligned}$$

Conjecture: $\tilde{y}_t = Az_t; \pi_t = Bz_t; i_t = \rho + \phi_\pi B z_t$

$$\begin{aligned}\tilde{y}_t &= A\rho_z z_t - \left(\phi_\pi B z_t - B\rho_z z_t - \left(1-\rho_z\right)z_t\right) \\ &= \left(A\rho_z - \phi_\pi B + B\rho_z + 1 - \rho_z\right)z_t \\ &\longrightarrow \quad A = \left(1-\rho_z\right)^{-1}\left(1-\rho_z + B\rho_z - \phi_\pi B\right)\end{aligned}$$

$$\pi_t = \left(\beta\rho_z B + \lambda(1+\varphi)A\right)z_t$$

$$\begin{aligned}\longrightarrow B &= \left(1-\beta\rho_z\right)^{-1}\lambda(1+\varphi)A \\ &= \left[\left(1-\beta\rho_z\right)\left(1-\rho_z\right)\right]^{-1}\left[\lambda(1+\varphi)\left(1-\rho_z + B\rho_z - \phi_\pi B\right)\right] \\ &= \lambda(1+\varphi)\frac{1-\rho_z + B\left(\rho_z - \phi_\pi\right)}{\left(1-\beta\rho_z\right)\left(1-\rho_z\right)} \\ &= \frac{\lambda(1+\varphi)}{1-\beta\rho_z} + \frac{\left(\rho_z-\phi_\pi\right)(1+\varphi)\lambda}{\left(1-\beta\rho_z\right)\left(1-\rho_z\right)}B\end{aligned}$$

$$
\begin{aligned}
&B=\frac{\lambda(1+\varphi)\left(1-\rho_z\right)}{\left(\rho_z-\varphi_\pi\right)(1+\varphi)\lambda+\left(1-\beta\rho_z\right)\left(1-\rho_z\right)}\\
&A=\frac{\left(1-\beta\rho_z\right)\left(1-\rho_z\right)}{\left(\rho_z-\varphi_\pi\right)(1+\varphi)\lambda+\left(1-\beta\rho_z\right)\left(1-\rho_z\right)}\\
&i_t=\rho+\phi_\pi B z_t\\
&r_t=i_t-\mathbb{E}_t\left(\pi_{t+1}\right)=i_t-\rho_z B z_t=p+\left(\phi_\pi-\rho_z\right)B z_t
\end{aligned}
$$

(e) Preference shock leads to expansion since $A>0$, and $B>0$. y y_t, π_t, i_t, and r_t go up. CB reacts by adjusting interest rate upward ($\phi_\pi>1$).

Monetary policy shock meas higher i_i and r_t. At the same time, y_t, and π_t decrease. Why though?

(f) By divine coincidence $i_t=\beta+\phi_\pi\pi_t$ with $\phi_\pi>1$ is sufficient to stabilize prices.

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Exercise 1

(1)

$$
\begin{aligned}
Y_t(i) &= C_t(i) + \gamma_t Y_t(i) \\
(1-\gamma_t)\, Y_t(i) &= C_t(i) \\
(1-\gamma_t) \int_0^1 Y_t(i) di &= \int_0^1 C_t(i) di \\
(1-\gamma_t)\, Y_t &= C_t \\
\ln(1-\gamma_t) + y_t &= c_t \\
y_t &= c_t - \ln(1-\gamma_t) = c_t - g_t
\end{aligned}
$$

(2) Optimality conditions of household are unchanged as $G_t(i)$ is financed via lump-sum:

$$
\begin{aligned}
C_t(i) &= \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \\
Q_t &= \mathbb{E}_t\left[\frac{C_t}{C_{t+1}}\frac{P_t}{P_{t+1}}\right] \\
C_t N_t^{\varphi} &= \frac{W_t}{P_t} \\
\frac{M_t}{P_t} &= C_t\left[1-\exp(-i_t)\right]^{-1} = (Y_t - G_t)\left[1-\exp(-i_t)\right]^{-1}
\end{aligned}
$$

Since Y_t increases in N_t, we have that a higher G_t must increase employment. Or C_t goes down?

Firms' pricing: (Assume $Y_t(i) = N_t(i)$)

$$
\max_{P_t(i)} \mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\frac{Y_t(i)}{P_t}\left(P_t(i) - W_t\right)\right]
$$

Use the market clearing condition: $Y_t(i) = C_t(i) + G_t(i)$

$$\max_{P_t(i)} \mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\frac{C_t(i)+G_t(i)}{P_t}\left(P_t(i)-W_t\right)\right]$$
$$\max_{P_t(i)} \mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\left(\frac{1}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}C_t+\frac{G_t(i)}{P_t}\right)\left(P_t(i)-W_t\right)\right]$$

$$\max_{P_t(i)} \mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\left(\frac{1}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}\left(Y_t-G_t\right)+\frac{G_t(i)}{P_t}\right)\left(P_t(i)-W_t\right)\right]$$

FOC:

$$\mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\frac{1}{P_t^{1-\varepsilon}}(1-\varepsilon)P_t(i)^{-\varepsilon}\left(Y_t-G_t\right)\big(1-P_t(i)^{-1}(-\varepsilon)W_t\big)+\frac{G_t(i)}{P_t(1-\varepsilon)}\right]=0$$
$$\mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\underbrace{\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}\frac{(Y_t-G_t)}{P_t}}_{\frac{Y_t(i)-G_t(i)}{P_t}}\big(1-P_t(i)^{-1}MW_t\big)+\frac{G_t(i)}{P_t(1-\varepsilon)}\right]=0$$
$$\mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\frac{Y_t(i)-G_t(i)}{P_t}\left(P_t(i)-MW_t\right)+\frac{G_t(i)}{P_t(1-\varepsilon)}\right]=0$$

Impose symmetry: $Y_t(i)=Y_t$ and $P_t(i)=P_t \quad \forall i$

$$\longrightarrow G_t(i)=G_t \text{ and } G_t(i)=C_t$$

$$\mathbb{E}_{t-1}\left[\Lambda_{t-1,t}\left(Y_t-G_t\right)\left(1-M\frac{W_t}{P_t}\right)+\frac{G_t}{P_t(1-\varepsilon)}\right]=0$$

Recall: $\Lambda_{t-1,t}=\beta\frac{C_{t-1}}{C_t}$

$$\mathbb{E}_{t-1}\left[(Y_{t-1}-G_{t-1})\left(1-M\frac{W_t}{P_t}\right)+\frac{G_t}{P_t(1-\varepsilon)}\right]=0$$
$$\mathbb{E}_{t-1}\left[\left(1-\gamma_{t-1}\right)Y_{t-1}\left(1-M\left(1-\gamma_t\right)N_t^{1+\varphi}\right)+\frac{\gamma_t Y_t}{P_t(1-\varepsilon)}\right]=0$$

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Exercise 1

(a) True

The DIS is given by

$$\tilde{y}_t = \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - r_t^n\right) + \mathbb{E}_t\left(\tilde{y}_{t+1}\right)$$

The CB can influence i_t which indirectly has an effect on $\mathbb{E}_t\left(\pi_{t+1}\right)$ as long as current and future inflation are linked. Even more so if the CB can commit to a policy.

(b) False

They depend on the nominal rate. And the nominal rate depends on inflation.

(c) False

Output increases but real wages actually increase since $\omega_t = w_t - p_t = \sigma c_t + n_t$ and in equilibrium $c_t = y_t$. Therefore, higher y_t implies higher real wages. (reference: slide deck 3, p. 37)

(d) True

Unless there are ”menu costs”. Output gap fluctuations still cause welfare losses.

Exercise 2

(a) u_t represents an exogenous shock that affects marginal costs. E.g. a pandemic causing supply-chains to break down. Since u_t follows on AR (1) process, the shocks are persistent over time.

The addition of u_t in this model ”breaks” the divine coincidence: the CB cannot stabilize output by only focusing on stabilizing prices.

(b) Conjectures:

$$\pi_t = \psi_\pi u_t \tag{I}$$
$$x_t = \psi_x u_t \tag{II}$$

Plug (I), (II) & interest rate rule into (1) & (2):

$$\left| \begin{aligned} &\psi_\pi u_t = \beta \mathbb{E}_t \left(\psi_\pi v_{t+1}\right) + \kappa \psi_x u_t + u_t \\ &\psi_x u_t = \mathbb{E}_t \left(\psi_x v_{t+1}\right) - \left(\Phi_\pi \psi_\pi u_t - \mathbb{E}_t \left(\psi_\pi u_{t+1}\right)\right) \end{aligned} \right|$$
$$\Leftrightarrow \left| \begin{aligned} &\psi_\pi = \beta \psi_\pi \rho_u + \kappa \psi_x + 1 \\ &\psi_x = \psi_x \rho_u - \phi_\pi \psi_\pi + \psi_\pi \rho_u \end{aligned} \right|$$
$$\Leftrightarrow \left| \begin{aligned} &\psi_\pi \left(1 - \beta \rho_u\right) = \kappa \psi_x + 1 \\ &\psi_x \left(1 - \rho_u\right) = \left(\rho_u - \phi_\pi\right) \psi_\pi \end{aligned} \right.$$
$$\Longrightarrow \psi_\pi \left(1 - \beta \rho_u\right) = \kappa \frac{\rho_u - \phi_\pi}{1 - \rho_u} \psi_\pi + 1$$
$$\Leftrightarrow \psi_\pi \left(1 - \beta \rho_u - \kappa \frac{\rho_u - \phi_\pi}{1 - \rho_u}\right) = 1$$
$$\pi_t = \frac{1 - \rho_u}{\left(1 - \beta \rho_u\right)\left(1 - \rho_u\right) - \kappa \left(\rho_u - \varnothing_\pi\right)} u_t$$
$$x_t = \frac{\rho_u - \phi_\pi}{\left(1 - \beta \rho_u\right)\left(1 - \rho_u\right) - \kappa \left(\rho_u - \varnothing_\pi\right)} u_t$$
$$\operatorname{Var}\left(x_t\right) = \psi_x^2 \operatorname{Var}\left(u_t\right)$$
$$\Rightarrow \frac{\partial \operatorname{Var}\left(x_t\right)}{\partial \phi_\pi} < 0$$

Volatility increases, the weaker CB reacts to π_t. Because divine coincidence is broken!

No (c) or (d)?

(e)

$$\begin{aligned}\min_{\phi_\pi} L(\phi_\pi) &= \mathrm{Var}(\pi_t) + \vartheta\,\mathrm{Var}(x_t)\\ &= \left(\psi_\pi^2 + \vartheta\psi_x^2\right)\mathrm{Var}(u_t)\end{aligned}$$

FOC:

$$0 = 2\psi_\pi \frac{\partial\psi_\pi}{\partial\phi_\pi} + 2\vartheta\psi_x\frac{\partial\psi_x}{\partial\phi_\pi}$$

$$\begin{aligned}0 = &(1-\rho_u)\left(-(1-\rho_u)\kappa\right)\\ &+\vartheta(\rho_u-\phi_\pi)\left(\kappa(\rho_u-\phi_\pi)-(1-\beta\rho_u)(1-\rho_u)-(\rho_u-\phi_\pi)\kappa\right)\\ &(1-\rho_u)^2\kappa = -\vartheta(\rho_u-\phi_\pi)(1-\beta\rho_u)(1-\rho_u)\\ &\rho_u-\phi_\pi = \frac{1-\rho_u}{1-\beta\rho_u}\frac{\kappa}{\vartheta}\\ &\phi_\pi = \rho_u + \frac{\kappa}{\vartheta}\frac{1-\rho_u}{1-\beta\rho_u}\end{aligned}$$

Note, that $\rho_u = 0$ implies $\phi_\pi = \frac{\kappa}{\theta}$.

At the same time $\rho_u = 1$ implies $\phi_\pi = 1$.

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Exercise 1

(1) u_t is what we call a cost-push-shock, and it is exogenous. Say cost increases due to an unpredictable shock (e.g. Covid). All else equal, p_t increases & thus inflation increases.

Usually: $u_t = (\hat{y}_t^e - \hat{y}_t^n)\,\kappa$

Divine coincidence is broken due to u_t influencing π_t independently of output.

(2) In class we found that

$$r_t^e = \rho + \sigma \mathbb{E}_t \left(y_{t+1}^e - y_t^e \right) = \rho + \mathbb{E}_t \left(y_{t+1}^e - y_t^e \right)$$

Therefore, we conclude that ε_t is the expected change in the efficient output. Since $\mathbb{E}\left(\varepsilon_t\right) = 0$, we have that y_t^e is also white noise. Additionally, ε_t captures anything that influences the efficient output.

(3) Conjecture:

$$\begin{aligned} \pi_t &= \psi_\pi u_t + \delta_\pi \varepsilon_t \\ x_t &= \psi_x u_t + \delta_x \varepsilon_t \end{aligned}$$

Plug conjectures in:

$$\begin{aligned} \psi_\pi u_t + \delta_\pi \varepsilon_t &= \beta \cdot 0 + \kappa \left(\psi_x u_t + \delta_x \varepsilon_t \right) + u_t \\ &= \left(\kappa \psi_x + 1 \right) u_t + \kappa \delta_x \varepsilon_t \\ \psi_x u_t + \delta_x \varepsilon_t &= 0 - \left(\phi_\pi \left(\psi_\pi u_t + \delta_\pi \varepsilon_t \right) - 0 - \varepsilon_t \right) \\ &= -\phi_\pi \psi_\pi u_t + \left(-\phi_\pi \delta_\pi + 1 \right) \varepsilon_t \end{aligned}$$

Therefore:

$$
\begin{aligned}
\psi_\pi &= \kappa\psi_x + 1 \\
\psi_x &= -\phi_\pi\psi_\pi \\
\implies (\psi_\pi, \psi_x) &= \left(\frac{1}{1+\kappa\phi_\pi}, \frac{-\phi_\pi}{1+\kappa\phi_\pi}\right)
\end{aligned}
$$

$$
\begin{aligned}
\delta_\pi &= \kappa\delta_x \\
\delta_x &= -\phi_\pi\delta_\pi + 1 \\
\implies (\delta_\pi, \delta_x) &= \left(\frac{\kappa}{1+\kappa\phi_\pi}, \frac{1}{1+\kappa\phi_\pi}\right)
\end{aligned}
$$

Together, we obtain that:

$$
\begin{aligned}
\pi_t &= \frac{1}{1+\kappa\phi_\pi}u_t + \frac{\kappa}{1+\kappa\phi_\pi}\varepsilon_t \\
x_t &= \frac{-\phi_\pi}{1+\kappa\phi_\pi}u_t + \frac{1}{1+\kappa\phi_\pi}\varepsilon_t
\end{aligned}
$$

(4)

$$
\begin{aligned}
&\min_{\phi_\pi} \operatorname{Var}(x_t) + \vartheta \operatorname{Var}(\pi_t) \\
&\min_{\phi_\pi} \left[\left(\frac{-\phi_\pi}{1+\kappa\phi_\pi}\right)^2 + \left(\frac{1}{1+\kappa\phi_\pi}\right)^2 \vartheta\right]\sigma_u^2 + \left[\left(\frac{-1}{1+\kappa\phi_\pi}\right)^2 + \left(\frac{-\kappa}{1+\kappa\phi_\pi}\right)^2 \upsilon\right]\sigma_\varepsilon^2 \\
&\min_{\phi_\pi} \frac{1}{(1+\kappa\phi_\pi)^2}\left[\left(\phi_\pi^2 + \vartheta\right)\sigma_u^2 + \left(1+\kappa^2\vartheta\right)\sigma_\varepsilon^2\right]
\end{aligned}
$$

FOC:

$$
\begin{aligned}
&2\phi_\pi\sigma_u^2(1+\kappa\phi_\pi)^2 - 2(1+\kappa\phi_\pi)\kappa\left[\left(\phi_\pi^2+\vartheta\right)\sigma_u^2 + \left(1+\kappa^2\vartheta\right)\sigma_\varepsilon^2\right] = 0 \\
&\iff \phi_\pi + \kappa\phi_\pi^2 = \kappa\phi_\pi^2 + \vartheta\kappa + \left(1+\kappa^2\vartheta\right)\frac{\sigma_\varepsilon^2}{\sigma_u^2}\kappa \\
&\iff \phi_\pi = \vartheta\kappa + \kappa\left(1+\kappa^2\vartheta\right)\frac{\sigma_\varepsilon^2}{\sigma_u^2}
\end{aligned}
$$

As cost-push-shocks become arbitrarily small ($\sigma_u^2 \to 0$) ϕ_π explodes since reacting to inflation is more and more important. This means inflation stabilization would be sufficient to stabilize output, i.e. the divine coincidence would work again.

(5) Flexible prices: $\mu_t = \mu \quad \forall t$ and $x_t = 0 \quad \forall t$

$$\begin{aligned}
\longrightarrow \pi_t &= \beta \mathbb{E}_t\left(\pi_{t+1}\right) + u_t ; \mathbb{E}_t\left(\pi_{t+1}\right) = \phi_\pi \pi_t - \varepsilon_t \\
\longrightarrow \pi_t &= \beta \phi_\pi \pi_t - \beta \varepsilon_t + u_t \\
\pi_t &= \frac{1}{1-\beta\phi_\pi} v_t - \frac{\beta}{1-\beta\phi_\pi} \varepsilon_t
\end{aligned}$$

Nope. Forget monetary policy, solve model. Plug in Taylor-rule etc.

5 Macroeconomics Final 2018 / 19

Exercise 1

(a) False

We showed in class that in this case, the optimal interest rate is zero. Together with the result of the representative consumer behaviour:

$$\pi = i_t - \rho = -\rho$$

I.e. constant deflation at rate ρ.

(b) False

We analyzed the following welfare loss:

$$W = -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C}$$

And looked at a 2nd order Taylor approximation:

$$W \approx \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(-\Phi \hat{x}_t + \left(G + \frac{\varphi + \alpha}{1 - \alpha} \right) \hat{x}_t^2 + \frac{\varepsilon}{\lambda} \pi_t^2 \right) + t.i.\rho.$$

Note, that π_t does enter the welfare loss such that $\pi_t = 0$ would lead to minimal welfare losses.

(c) True

We have seen that with a distortional labour income tax, the divine coincidence breaks down. This means, inflation targeting is insufficient to stabilize outcome.

More specifically in exercise 2, PS3, we found the following NKPC:

$$\pi_t = \beta \mathbb{E}_t (\pi_{t+1}) + \lambda (1 + \varphi) x_t + \lambda (\mu + \tau_t)$$

(d) True.

It is only attainable if the natural real wage does not change, i.e. is constant.

Exercise 2

(a)

$$\max_{N_t} (P_t - 1) N_t$$

This implies

$$N_t = \begin{cases} \infty & \text{if} \quad P_t > 1 \\ \mathbb{R}^+ & \text{if} \quad P_t = 1 \\ 0 & \text{if} \quad P_t < 1 \end{cases}$$

Therefore, we may conclude that $P_t = 1$, and the firm employs everyone that wants to work.

(b) First, obtain labour supply by solving

$$\begin{aligned} &\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right] \\ &\text{s.t. } 0 = B_{t-1} + M_{t-1} + N_t - P_t C_t - Q_t B_t - M_t \end{aligned}$$

Classic result from FOCs:

$$\begin{aligned} N_t^{\varphi} &= \frac{W_t}{P_t} = 1 \\ \Rightarrow N_t &= 1 \end{aligned}$$

Thus, labour supply is 1 , and this is the maximum that can be produced. Use CIA & market clearing:

$$G_t = Y_t = \frac{M_t}{Z_t}$$

We conclude that:

$$Y_t = \min\left\{\frac{M_t}{Z_t}, 1\right\}$$

(c) The positive shock impacts the CIA constraint. Increase in money demanded per unit of consumption. If supply does not increase (e.g. because $N_t = 1$ is hit), need to reduce consumption, and lower output.

(d) Social planner problem:

$$\max \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(C_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\right]$$
$$\text{s.t. } C_t = Y_t = N_t$$

Plug constraint into objective function:

$$\max_{N_t} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(N_t - \frac{N_t^{1+\varphi}}{1+\varphi}\right)\right]$$

FOC:

$$1 - N_t^{\varphi} = 0 \Leftrightarrow N_t = 1$$

By production technology:

$$Y_t^e = 1$$

Use $Y_e = 1, P_t = 1$ in CIA constraint:

$$M_t = Z_t$$

I.e. the money supply reacts to Z_t.

(e) No. Both short- and long-run $Y_t = \min\left\{\frac{M_t}{Z_t}, 1\right\}$.

Here, prices and wages are fixed, and there is a persistent effect of money on output.

In NK model, both prices & wages can be adjusted.

6 Macroeconomics Final 2019 / 20

Exercise 1

(a) False.

In this model $\Delta m = \pi$, i.e. the CB controls inflation by printing money only. Additionally, the social planner's problem yields $\pi_t = -r_{t-1}$. Therefore, as long as real interest is positive, money is not growing but declining.

(b) False.

Demand shocks do not matter, but cost-push shocks for example destroy the divine coincidence.

Thus, the CB should also be targeting output gap.

(c) True.

In the model, output spikes at t and goes back slowly. In reality, the output increases slowly & stays at a higher level. Employment drops directly at t both in the model and in the data. But the model predicts that it rises back to previous level and in the data employment settles at a higher level.

(d) False.

Inflation should move with output gap. But one should not define output gap as detrended GDP because the trend is not necessarily the natural output.

Exercise 2

(a) (1) Derive optimal expenditure allocation

$$\min_{\{C_t(i)\}} \int_0^1 P_t(i)C_t(i)di$$
$$\text{s.t.} \quad \left[\int_0^1 C_t(i)^{1-1/\varepsilon} di\right]^{\frac{\varepsilon}{\varepsilon-1}} = C_t$$
$$\Rightarrow \int_0^1 P_t(i)C_t(i)di = P_tC_t$$

(2) Solve consumer's problem:

$$\max \mathbb{E}_0 \left\{\sum_{t=0}^{\infty} \beta^t \left(\ln\left(C_t\right) - N_t\right) Z_t\right\} \text{ where } C_t = \left[\int_0^1 C_t(i)^{1-\frac{i}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \ , \varepsilon > 1$$
$$\text{s.t. } P_tC_t + Q_tB_t + M_t \leq B_{t-1} + M_{t-1} + W_tN_t(1+\tau) + P_tD_t - P_tT_t$$

Optimality Conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = C_t = \frac{W_t}{P_t}(1+\tau)$$
$$Q_t = \beta \mathbb{E}_t \left[\frac{U_{c,t+1}}{U_{ct}} \frac{P_t}{P_{t+1}}\right] = \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}}\right]$$
$$M_t = P_tC_t$$

Log-linearize the optimality conditions and recall that $Q_t = \exp\left(-i_t\right)$ and $\beta = \exp(-\rho)$:

$$w_t - p_t = c_t - \ln(1+\tau)$$
$$c_t = \mathbb{E}_t\left(c_{t+1}\right) - \left(i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \rho\right) - \mathbb{E}_t\left(z_{t+1}\right) + z_t$$
$$m_t = p_t + c_t$$

(b) Markup:

$$
\begin{aligned}
\mu_t &= p_t - w_t + a_t \text{ (because CRS)} \\
&= \tau - y_t + a_t
\end{aligned}
$$

Under flexible prices:

$$
\begin{aligned}
\mu &= \tau - y_t^n + a_t \\
\Leftrightarrow y_t^n &= \tau - \mu + a_t \quad \text{(natural output)} \\
\Leftrightarrow n_t^n &= y_t^n - a_t = \tau - \mu \quad \text{(natural employment)}
\end{aligned}
$$

Social planner's problem:

$$
\begin{aligned}
&\max \left(\ln \left(C_t\right) - N_t\right) Z_t \\
&\text{s.t. } C_t = A_t N_t \\
\Longrightarrow &\max_{N_t} \left(\ln \left(A_t N_t\right) - N_t\right) Z_t
\end{aligned}
$$

FOC:

$$
\frac{1}{N_t} - 1 = 0 \Leftrightarrow N_t = 1
$$

As a result:

$$
\begin{aligned}
y_t^e &= a_t \quad \text{(efficient output)} \\
n_t^e &= 0 \quad \text{(efficient employment)}
\end{aligned}
$$

(c) Dynamic IS:

$$
\begin{aligned}
c_t &= \mathbb{E}_t\left(c_{t+1}\right) - \left(i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \rho\right) - \mathbb{E}_t\left(z_{t+1}\right) + z_t && \text{(Euler Equ)} \\
c_t &= y_t && \text{(market)} \\
z_{t+1} &= \rho z_t + \nu_{t+1} && \text{(AR(1))}
\end{aligned}
$$

$$
\begin{aligned}
y_t &= \mathbb{E}\left(y_{t+1}\right) - \left(i_t - \mathbb{E}\left(\pi_{t+1}\right) - \rho\right) + \left(1-\rho_z\right) z_t \\
y_t - y_t^n &= \mathbb{E}\left(y_{t+1}\right) - \left(i_t - \mathbb{E}\left(\pi_{t+1}\right) - \rho\right) + \left(1-p_z\right) z_t - y_t^n + \mathbb{E}\left(y_{t+1}^n - y_{t+1}^n\right) \\
\tilde{y}_t &= \mathbb{E}\left(\tilde{y}_{t+1}\right) - \left(i_t - \mathbb{E}\left(\pi_{t+1}\right) - \rho\right) + \left(1-\rho_z\right) z_t + \mathbb{E}\left(\Delta y_{t+1}^n\right) \\
\tilde{y}_t &= \mathbb{E}\left(\tilde{y}_{t+1}\right) - \left(i_t - \mathbb{E}\left(\pi_{t+1}\right) - \rho\right) + \left(1-\rho_z\right) z_t - \left(1-\rho_a\right) a_t \text{ (DIS)}
\end{aligned}
$$

<u>NKPC:</u>

Firms:

$$
\begin{aligned}
Y_t(i) &= A_t N_t \quad \text{(technology CRS)} \\
\psi_t(i) &= w_t - a_t \quad \text{(marginal cost)}
\end{aligned}
$$

Firms can reset prices every period with probability $(1-\theta)$. The following price level dynamics emerge:

$$
\begin{aligned}
P_t &= \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad (1)
\end{aligned}
$$

In SS: $P_t = P_{t-1} = P_t^*$

Thus log-lin around SS of (1):

$$
p_t = \theta p_{t-1} + (1-\theta) p_t^*
$$

Optimal price setting:

$$
\begin{aligned}
&\max_{P_t^*} \sum_{t=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t,t+k} \frac{1}{P_{t+k}} \left(P_t^* Y_{t+k|t} - C_{t+k} \left(Y_{t+k|t} \right) \right) \right] \\
&\text{s.t.} \quad Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}
\end{aligned}
$$

Optimality condition:

$$
\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left[\Lambda_{t+t|k} Y_{t+k|t} \frac{1}{P_{t+k}} \left(P_t^* - M\psi_{t+k|t} \right) \right] = 0
$$

log-linearize the result:

$$
\begin{aligned}
p_t^* &= \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_t \left(\psi_{t+k|t} \right) \\
&\vdots \\
p_t^* &= \beta\theta \mathbb{E}_t \left(p_{t+1}^* \right) + (1-\beta\theta) p_t - (1-\beta\theta) \left(\mu_t - \mu \right) \\
p_t - \theta p_{t-1} &= (1-\theta) \left[\beta\theta \mathbb{E}_t \left(p_{t+1}^* \right) + (1-\beta\theta) p_t - (1-\beta\theta) \left(\mu_t - \mu \right) \right] \\
\theta p_t + (1-\theta) p_t - \theta p_{t-1} &= (1-\theta) \left[\beta\theta \mathbb{E}_t \left(p_{t+1}^* \right) + (1-\beta\theta) p_t - (1-\beta\theta) \left(\mu_t - \mu \right) \right] \\
\theta \left(p_t - p_{t-1} \right) &= (1-\theta) \left[\beta\theta \mathbb{E}_t \left(p_{t+1}^* \right) - \beta\theta p_t - (1-\beta\theta) \left(\mu_t - \mu \right) \right] \\
\pi_t &= (1-\theta) \beta \mathbb{E}_t \left(p_{t+1}^* \right) - (1-\theta) \beta p_t - \frac{(1-\beta\theta)(1-\theta)}{\theta} \left(\mu_t - \mu \right)
\end{aligned}
$$

Note: $(1-\theta)$ is the part of firms that change prices. This determines inflation:

I think, starting here would have been fine as well:

$$\begin{aligned}
\pi_t &= \beta\mathbb{E}_t\left(\pi_{t+1}\right) - \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\mu_t - \mu\right) && (2)\\
\mu_t &= \ln(1+\tau) - y_t - a_t\\
\mu &= \ln(1+\tau) - y_t^n - a_t\\
\Rightarrow \mu_t - \mu &= -\tilde{y}_t && (3)
\end{aligned}$$

Combine (2) & (3):

$$\pi_t = \beta\mathbb{E}_t\left(\pi_{t+1}\right) + \frac{(1-\beta\theta)(1-\theta)}{\theta}\tilde{y}_t \quad (NKPC)$$

(d) $r_t^n = \rho - (1-\rho_z)\, z_t + (1-\rho_a)\, a_t$ directly from DIS

(e) This implies $y_t^n = y_t^e$. Thus $\tilde{y}_t$ is the welfare relevant output gap. Make $\tilde{y} = 0$ to minimize welfare losses. Using this in DIS & NKPC:

$$\begin{aligned}
i_t &= \mathbb{E}\left(\pi_{t+1}\right) + \rho + (1-\rho_z)\, z_t - (1-\rho_a)\, a_t\\
\pi_t &= \beta\mathbb{E}_t\left(\pi_{t+1}\right)\\
\longrightarrow \quad i_t &= \beta^{-1}\pi_t + \rho + (1-\rho_z)\, z_t - (1-\rho_a)\, a_t\\
&= \phi_\pi \pi_t + r_t^n \quad \text{with } \phi_\pi > 1 \text{ as in lecture}\\
n_t &= 0; \quad y_t = a_t \longrightarrow \tilde{y}_t = 0
\end{aligned}$$

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Exercise 1

(a) False

Expansionary shock lowers the interest rate, and increases the output. But as the labour supply is defined as

$$w_t - p_t = \sigma c_t + n_t \text{ and } c_t = y_t$$

Higher y_t leads to higher real wages.

(b) True

Inflation targeting is optimal when wages are flexible as this stabilized output at the same time by the "divine coincidence". Under sticky wages, this breaks down & the central bank should also aim to stabilize output ($ unemployment).

(c) False ?

In the classical model, prices react to money growth more than 1:1. While prices react much less to money growth, as they are sticky, they still react, albeit to a lesser degree.

(d) False

Because some firms are not able to adjust their prices, the inflation from last period is somewhat predictive for current inflation.

Exercise 2

Market clearing:

$$
\begin{aligned}
Y_t &= C_t + \gamma_t Y_t \\
\Leftrightarrow Y_t &= \frac{1}{1-\gamma_t} C_t = F_t C_t \\
y_t &= f_t + c_t
\end{aligned}
$$

HH optimality:

$$
\begin{aligned}
N_t C_t &= \frac{W_t}{P_t} \\
\Longrightarrow n_t + c_t &= w_t - p_t \\
Q_t &= \beta \mathbb{E}_t \left\{ \frac{C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} \\
\Rightarrow i_t &= \beta + \mathbb{E}_t \left(c_{t+1} \right) - c_t + \mathbb{E} \left(\pi_{t+1} \right)
\end{aligned}
$$

Firms:

$$
\begin{aligned}
P_t &= \psi_t M_t = W_t M_t \\
\Rightarrow p_t &= \mu_t + w_t
\end{aligned}
$$

(a) μ is constant:

$$
\begin{aligned}
\mu &= p_t - w_t = -\left(n_t + c_t \right) \\
&= -\left(y_t + y_t - f_t \right) = f_t - 2y_t \\
\longrightarrow y_t^n &= \frac{1}{2} \left(f_t - \mu \right)
\end{aligned}
$$

Since y_t^n rises in f_t, we conclude that γ_t increases output. Note, that the increase is not $1:1$ as g_t crowds out some of the consumption.

(b) By market clearing:

$$Y_t = G_t + C_t = F_t C_t \quad (\text{since } G_t = \gamma_t Y_t \ )$$

In logs:

$$y_t = f_t + c_t \tag{1}$$

Need to find DIS. Stat with EE, plug in c_t from (1):

$$\begin{aligned}
\underbrace{i_t}_{=\rho+\phi_y \hat{y}_t} &= \rho + \mathbb{E}_t\left(c_{t+1}\right) - c_t + \underbrace{\mathbb{E}\left(\pi_{t+1}\right)}_{=0 \text{ by const. prices}} \\
y_t - f_t &= \mathbb{E}_t\left(y_{t+1} - f_{t+1}\right) - \phi_y \hat{y}_t
\end{aligned}$$

Express in deviations from SS. By $\gamma_t \sim iid(\gamma)$, the expected deviation from SS is zero.

$$\begin{aligned}
\hat{y}_t &= \mathbb{E}_t\left(\hat{y}_{t+1}\right) + \hat{f}_t - \phi_y \hat{y}_t \\
\hat{y}_t &= \frac{1}{1+\phi_y}\left(\mathbb{E}_t\left(\hat{y}_{t+1}\right) + \hat{f}_t\right)
\end{aligned}$$

Now, iterate forward, using $\mathbb{E}_t\left(\hat{f}_{t+1}\right) = 0$:

$$\begin{aligned}
\mathbb{E}_t\left(\hat{y}_{t+1}\right) &= \frac{1}{1+\phi_y}\mathbb{E}_t\left(\hat{y}_{t+2}\right) \\
&\vdots \\
\mathbb{E}_t\left(\hat{y}_{t+1}\right) &= \left(\frac{1}{1+\phi_y}\right)^j \mathbb{E}_t\left(\hat{y}_{t+j+1}\right) \xrightarrow{j\to\infty} 0
\end{aligned}$$

Thus, we are left with

$$\hat{y}_t = \frac{1}{1+\phi_y}\hat{f}_t$$

Channel: Increase in aggregate demand for goods which is met by increased production.

(c) Generally:

$$\frac{dY_t}{dG_t} = \frac{dY_t/d\gamma_t}{dG_t/d\gamma_t} = \frac{dY_t/d\gamma_t}{d\left(\gamma_t Y_t\right)/d\gamma_t} = \frac{dY_t/d\gamma_t}{Y_t + \gamma_t\left(dY_t/d\gamma_t\right)}$$

In (a):

$$\begin{aligned} y_t^n &= \frac{1}{2}\left(f_t - \mu\right) \Rightarrow y_t^n = \left[F_t M\right]^{1/2} = \left(1-\gamma_t\right)^{-1/2} M^{1/2} \\ dY_t/d\gamma_t &= -\frac{1}{2}\left(1-\gamma_t\right)^{-3/2}(-1) M^{1/2} = \frac{Y_t^n}{2\left(1-\gamma_t\right)} \\ \Rightarrow \frac{dY_t}{dG_t} &= \frac{Y_t^n}{2\left(1-\gamma_t\right)Y_t^n + Y_t^n\gamma_t} = \frac{1}{2-\gamma_t} \end{aligned}$$

In (b):

$$dY_t/d\gamma_t = Y\frac{dy_t}{df_t}\frac{df_t}{d\gamma_t} = Y\frac{1}{1+\phi_y}\frac{1}{1-\gamma} = \frac{1}{1+\phi_y(1-\gamma)}$$

(d) In (a) mandatory policy is neutral (no effect of ϕ_y on Y_t^n).

In (b), if monetary policy is more countercyclical (ϕ_y bigger), the spending multiplier is smaller. Reason: effect of government spending is partly offset by the rise in the interest rate.

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Exercise 1

(a) False

We saw, that the optimal monetary policy is $i_t = 0 \quad \forall t$. Therefore, a constant deflation at rate ρ would be optimal.

(b) True

We saw that the divine coincidence breaks down in this type of economy. Therefore, only targeting price inflation is not optimal.

CB should also target wage inflation and/or output gap.

(c) Uncertain

The interest rate was set consistently too low compared to the Taylor rule, using current data. But when looking at the data, the Fed had at that point in time, the interest rate is very close to the behaviour predicted by the Taylor rule.

(d) True

In the short run, this is always the case. Note however that in the case of flexible prices & sticky wages, price inflation will be lower (compared to baseline) after one period.

Exercise 2

(a) Let the representative consumer solve

$$\max_{C_t,N_t,B_t,M_t} \mathbb{E}_0 \left(\sum_{t=0}^{\infty} \beta^t \left(\ln \left(C_t \right) - V \left(N_t \right) \right) \xi_t \right)$$

$$\text{s.t. } P_t C_t + Q_t B_t + M_t = M_{t-1} + B_{t-1} + W_t N_t + P_t D_t + P_t T_t$$

$$M_t = \int P_t(i) C_t(i) di$$

$$P_t C_t = \int P_t(i) C_t(i) di$$

Then the Euler equation will be

$$Q_t = \beta \mathbb{E}_t \left[\frac{U_{C,t+1}}{U_C, t} \frac{P_t}{P_{t+1}} \right] = \beta \mathbb{E}_t \left[\frac{C_t \xi_t}{C_{t+1} \xi_{t+1}} \Pi_{t+1}^{-1} \right]$$

Log-linearized using $Q_t = \exp\left(-i_t\right); \beta = \exp(-\rho)$:

$$c_t = \mathbb{E}_t \left\{ c_{t+1} \right\} - \left(i_t - \mathbb{E}_t \left(\pi_{t+1} \right) - \rho \right) + \underbrace{\ln \left[\mathbb{E}_t \left\{ \frac{\xi_{t+1}}{\xi_t} \right\} \right]}_{\equiv z_t \text{ iid}(0,\sigma_z^2)}$$

$$\Leftrightarrow c_t = \mathbb{E}_t \left\{ c_{t+1} \right\} - \left(i_t - \mathbb{E}_t \left(\pi_{t+1} \right) - \rho - z_t \right)$$

By market clearing: $c_t \stackrel{!}{=} y_t$

Also let $\tilde{y}_t = y_t - y_t^n$, then

$$\tilde{y}_t = \mathbb{E}_t \left\{ \tilde{y}_{t+1} \right\} - \left(i_t - \mathbb{E}_t \left(\pi_{t+1} \right) - r_t^n \right)$$

where $r_t^n = \rho + z_t = \rho_t$

Now $x = y_t - y_t^e = y_t - y_t^n - (y_t^e - y_t^n)$

Assume that $y_t^n = y_t^e$, i.e. the natural output is efficient, then we end up with:

$$x_t = \mathbb{E}_t \left\{ x_{t+1} \right\} - \left(i_t - \mathbb{E}_t \left\{ \pi_{t+1} \right\} - \rho - z_t \right)$$

Also, one must assume that EE can be log-linearized around SS, and that goods markets clear. Also, by $U(\cdot), \sigma = 1$, and here p_t is time-varying.

(b) Plug policy into (1):

$$x_t = \mathbb{E}_t \{x_{t+1}\} - (\phi_\pi \pi_t - \mathbb{E}_t \{\pi_{t+1}\} - z_t) \tag{I}$$

Conjecture:

$$\begin{aligned} \pi_t &= \delta_\pi z_t + \psi_\pi u_t \\ x_t &= \delta_x z_t + \psi_x u_t \end{aligned}$$

Conjectures into (I):

$$\begin{aligned} \delta_x z_t + \psi_x u_t =& \mathbb{E}_t \{\delta_x z_t + \psi_x u_t\} - \phi_\pi (\delta_\pi z_t + \psi_\pi u_t) \\ &+ \mathbb{E}_t \{\delta_\pi z_{t+1} + \psi_\pi u_{t+1}\} + z_t \\ =& - \phi_\pi \delta_\pi z_t - \phi_\pi \psi_\pi u_t + z_t \\ =& (1 - \phi_\pi \delta_\pi) z_t - \phi_\pi \psi_\pi u_t \end{aligned}$$

Therefore we have the intermediate results:

$$\delta_x = 1 - \phi_\pi \delta_\pi \tag{II}$$
$$\psi_x = -\phi_\pi \psi_\pi \tag{III}$$

Conjectures into (2):

$$\begin{aligned} \delta_\pi z_t + \psi_\pi u_t =& \beta \mathbb{E}_t \{\delta_\pi z_{t+1} + \psi_\pi u_{t+1}\} + \kappa (\delta_x z_t + \psi_x u_t) + u_t \\ =& \kappa \delta_x z_t + (\kappa \psi_x + 1) u_t \end{aligned}$$

Achieve the following intermediate results:

$$\delta_\pi = \kappa\delta_x \tag{IV}$$

$$\psi_\pi = \kappa\psi_x + 1 \tag{V}$$

Combine (II) & (IV):

$$\begin{aligned} &\delta_\pi = \kappa\left(1-\phi_\pi\delta_\pi\right) = \kappa - \kappa\phi_\pi\delta_\pi \\ \Leftrightarrow \quad &\delta_\pi = \frac{\kappa}{1+\kappa\phi_\pi} \Rightarrow \delta_x = \frac{1}{1+\kappa\delta_\pi} \end{aligned}$$

Combine (III) & (V):

$$\begin{aligned} \psi_\pi &= -\kappa\phi_\pi\psi_\pi + 1 \\ \psi_\pi &= \frac{1}{1+\kappa\phi_\pi} \Rightarrow \psi_x = \frac{-\phi_\pi}{1+\kappa\phi_\pi} \end{aligned}$$

Therefore, the equilibrium behaviour is

$$\begin{aligned} \pi_t &= \frac{\hat{\kappa}}{1+\kappa\phi_\pi} z_t + \frac{1}{1+\kappa\phi_\pi} u_t \\ x_t &= \frac{1}{1+\kappa\phi_\pi} z_t + \frac{-\phi_\pi}{1+\kappa\phi_\pi} u_t \end{aligned}$$

The welfare loss is then given by:

$$\begin{aligned} L = &\operatorname{Var}\left(\frac{\kappa}{1+\kappa\phi_\pi} z_t + \frac{1}{1+\kappa\phi_\pi} u_t\right) + \vartheta\operatorname{Var}\left(\frac{1}{1+\kappa\phi_\pi} z_t + \frac{-\phi_\pi}{1+\kappa\phi_\pi} u_t\right) \\ = &\left[\left(\frac{\kappa}{1+\kappa\phi_\pi}\right)^2 + \vartheta\left(\frac{1}{1+\kappa\phi_\pi}\right)^2\right]\sigma_z^2 + \left[\left(\frac{1}{1+\kappa\phi_\pi}\right)^2 + \vartheta\left(\frac{\phi_\pi}{1+\kappa\phi_\pi}\right)^2\right]\sigma_u^2 \\ = &\frac{\left(\kappa^2+\vartheta\right)\sigma_z^2 + \left(1+\vartheta\phi_\pi^2\right)\sigma_u^2}{\left(1+\kappa\phi_\pi\right)^2} \end{aligned}$$

(c) We solve

$$\min_{\phi_\pi} \frac{(\kappa^2+\vartheta)\,\sigma_z^2 + (1+\vartheta\phi_\pi^2)\,\sigma_u^2}{(1+\kappa\phi_\pi)^2}$$

FOC:

$$\begin{aligned}
0 &= \vartheta 2\phi_\pi\sigma_u^2 (1+\kappa\phi_\pi)^2 - 2(1+\kappa\phi_\pi)\,\kappa\left(\left(\kappa^2+\vartheta\right)\sigma_z^2 + \left(1+\vartheta\phi_\pi^2\right)\sigma_u^2\right) \\
0 &= \vartheta\phi_\pi\sigma_u^2 (1+\kappa\phi_\pi) - \kappa\left(\left(\kappa^2+\vartheta\right)\sigma_z^2 + \left(1+\vartheta\phi_\pi^2\right)\sigma_u^2\right)
\end{aligned}$$

$$\begin{aligned}
\vartheta\phi_\pi\sigma_u^2 (1+\kappa\phi_\pi) &= \kappa\left(\left(\kappa^2+\vartheta\right)\sigma_z^2 + \left(1+\vartheta\phi_\pi^2\right)\sigma_u^2\right) \\
\vartheta\phi_\pi\sigma_u^2 + \kappa\vartheta\phi_\pi^2\sigma_u^2 &= \kappa\left(\kappa^2+\vartheta\right)\sigma_z^2 + \kappa\sigma_u^2 + \kappa\vartheta\phi_\pi^2\sigma_u^2 \\
\vartheta\phi_\pi\sigma_u^2 &= \kappa\left(\kappa^2+\vartheta\right)\sigma_z^2 + \kappa\sigma_u^2 \\
\phi_\pi &= \frac{\kappa}{\vartheta}\left[\left(\kappa^2+\vartheta\right)\frac{\sigma_z^2}{\sigma_u^2} + 1\right]
\end{aligned}$$

$$\begin{aligned}
&\text{for } \sigma_u^2 \longrightarrow 0 : \phi_\pi \to \infty \\
&\text{for } \sigma_z^2 \longrightarrow 0 : \phi_\pi \longrightarrow \frac{\kappa}{\vartheta}
\end{aligned}$$

Intuition: Cost-push shocks going to zero, means CB reacts only to real interest rate shocks.

Since $\pi_t = i_t - \rho_t = i_t - \rho - z_t$, this means the CB reacts as hard as it con to changes in inflation. If interest rate shocks go to zero, the CB reacts to u_t directly by (2), and also indirectly by the equilibrium behaviour from (b).

Don't really understand this tbh.